# Contents

**Banach Gelfand Triples for Gabor Analysis** .................. 1
H. Feichtinger, F. Luef, and E. Cordero
1 Introduction ............................................. 1
2 Preliminaries ............................................ 3
3 Gabor Analysis on $\boldsymbol{L}^2$ ........................................ 6
4 Time–Frequency Representations ............................ 9
5 The Gelfand Triple $(\boldsymbol{S}_0, \boldsymbol{L}^2, \boldsymbol{S}_0')(\mathbb{R}^{\boldsymbol{d}})$ ........................ 12
6 The Spreading Function and Pseudo-Differential Operators ....... 19
7 Gabor Multipliers ......................................... 29
References .................................................. 31

**Four Lectures in Semiclassical Analysis for Non Self-Adjoint Problems with Applications to Hydrodynamic Instability** ..... 35
B. Helffer
1 General Introduction ...................................... 35
2 Lecture 1: The Rayleigh–Taylor Model ....................... 37
2.1 The Rayleigh–Taylor Model: Physical Origin ............... 37
2.2 Rayleigh–Taylor Mathematically .......................... 40
2.3 Elementary Spectral Theory .............................. 41
2.4 A Crash Course on $h$-Pseudodifferential Operators ........... 42
2.5 Application for Rayleigh–Taylor: Semi-Classical Analysis for $K(h)$ ............................................ 44
2.6 Harmonic Approximation ................................ 45
2.7 Instability of Rayleigh–Taylor: An Elementary Approach via WKB Constructions .................................... 46
3 Lecture 2: Towards Non Self-Adjoint Models .................. 49
3.1 Instability for Kelvin–Helmholtz I: Physical Origin .......... 49
3.2 Around the $\epsilon$-Pseudo-Spectrum ........................... 50
3.3 Around the $h$-Family-Pseudospectrum ...................... 51
3.4 The Davies Example by Hand ............................. 52

3.5 Kelvin–Helmholtz II: Mathematical Analysis ... 55
3.6 Other Toy Models ... 58
4 Lecture 3: On Semi-Classical Subellipticity ... 58
4.1 Introduction ... 58
4.2 Non Subellipticity: Generic Result ... 59
4.3 Link with the Standard Non-Hypoellipticity Results for Operators of Principal Type ... 60
4.4 Elementary Proof for the Non-Subelliptic Model ... 60
4.5 $\frac{1}{2}$ Semi-Classical Subellipticity ... 62
5 Lecture 4: Other Non Self-Adjoint Models Coming from Hydrodynamics ... 63
5.1 Introduction ... 63
5.2 Quasi-Isobaric Model (Kull and Anisimov) ... 65
5.3 Stationary Laminar Solution ... 65
5.4 From the Physical Parameters to the Relevant Mathematical Parameters ... 66
5.5 The Convection Velocity Model ... 67
5.6 The Model for the Ablation Regime ... 69
5.7 Semi-Classical Regimes for the Ablation Models ... 71
5.8 Subellipticity II: At the Boundary of $\Sigma(a_0)$ ... 73
References ... 75

**An Introduction to Numerical Methods of Pseudodifferential Operators** ... 79
M.P. Lamoureux and G.F. Margrave
1 Signal Processing and Pseudodifferential Operators ... 79
1.1 Introduction to Seismic Imaging ... 79
1.2 Introduction to Pseudodifferential Operators ... 82
1.3 A Jump in Dimension ... 87
1.4 Boundedness of the Operators ... 89
2 Manipulating Pseudodifferential Operators ... 93
2.1 Composition of Operators ... 93
2.2 Asymptotic Series ... 95
2.3 Oscillatory Integrals ... 96
2.4 Other Pseudo-Topics ... 99
3 Numerical Implementations ... 100
3.1 Sampling and Quantization Error in Signal Processing ... 100
3.2 The Discrete Fourier Transform and Periodization Errors ... 102
3.3 Direct Numerical Implementation via the DFT ... 103
3.4 Operations Count ... 106
3.5 Numerical Implementation via Product-Convolution Operators 107
3.6 Almost Diagonalization via Wavelet and Gabor Bases ... 108
4 Gabor Multipliers ... 110
4.1 Short Time Fourier Transforms and Their Multipliers ... 110
4.2 Gabor Transforms and Gabor Multipliers ... 113

5 Gabor Transforms in Practice . . . 116
5.1 Sampled Space . . . 116
5.2 Sampling in the Frequency Domain . . . 119
5.3 Partitions of Unity and Frequency Subsampling . . . 121
5.4 Uniform POUs . . . 126
6 Seismic Imaging . . . 130
6.1 Wavefield Extrapolation . . . 130
References . . . 132

**Some Facts About the Wick Calculus** . . . 135
N. Lerner
1 Elementary Fourier Analysis via Wave Packets . . . 135
1.1 The Fourier Transform of Gaussian Functions . . . 135
1.2 Wave Packets and the Poisson Summation Formula . . . 136
1.3 Toeplitz Operators . . . 140
2 On the Weyl Calculus of Pseudodifferential Operators . . . 141
2.1 A Few Classical Facts . . . 141
2.2 Symplectic Invariance . . . 143
2.3 Composition Formulas . . . 145
3 Definition and First Properties of the Wick Quantization . . . 147
3.1 Definitions . . . 147
3.2 The Gårding Inequality with Gain of One Derivative . . . 151
3.3 Variations . . . 152
4 Energy Estimates via the Wick Quantization . . . 156
4.1 Subelliptic Operators Satisfying Condition $(P)$ . . . 156
4.2 Polynomial Behaviour of Some Functions . . . 158
4.3 Energy Identities . . . 162
5 The Fefferman–Phong Inequality . . . 164
5.1 The Semi-Classical Inequality . . . 164
5.2 The Sjöstrand Algebra . . . 165
5.3 Composition Formulas . . . 166
5.4 Sketching the Proof . . . 167
5.5 A Final Comment . . . 172
6 Appendix . . . 172
6.1 Cotlar's Lemma . . . 172
References . . . 173

**Schatten Properties for Pseudo-Differential Operators on Modulation Spaces** . . . 175
J. Toft
1 Introduction . . . 175
2 Preliminaries . . . 178
3 Schatten–Von Neumann Classes for Operators Acting on Hilbert Spaces . . . 185

4 Schatten–Von Neumann Classes for Operators Acting on Modulation Spaces . . . . . 188
5 Continuity and Schatten–Von Neumann Properties for Pseudo-Differential Operators . . . . . 191
References . . . . . 201

**List of Participants** . . . . . 203