

# Contents

<b>List of Symbols</b> . . . . .	1
<b>1 Introduction</b> . . . . .	3
1.1 Problem Statements . . . . .	3
1.1.1 The Optimal Control Problem . . . . .	3
1.1.2 The Differential Game Problem . . . . .	4
1.2 Examples . . . . .	5
1.3 Static Optimization . . . . .	18
1.3.1 Unconstrained Static Optimization . . . . .	18
1.3.2 Static Optimization under Constraints . . . . .	19
1.4 Exercises . . . . .	22
<b>2 Optimal Control</b> . . . . .	23
2.1 Optimal Control Problems with a Fixed Final State . . . . .	24
2.1.1 The Optimal Control Problem of Type A . . . . .	24
2.1.2 Pontryagin's Minimum Principle . . . . .	25
2.1.3 Proof . . . . .	25
2.1.4 Time-Optimal, Frictionless, Horizontal Motion of a Mass Point . . . . .	28
2.1.5 Fuel-Optimal, Frictionless, Horizontal Motion of a Mass Point . . . . .	32
2.2 Some Fine Points . . . . .	35
2.2.1 Strong Control Variation and global Minimization of the Hamiltonian . . . . .	35
2.2.2 Evolution of the Hamiltonian . . . . .	36
2.2.3 Special Case: Cost Functional $J(u) = \pm x_i(t_b)$ . . . . .	36

2.3	Optimal Control Problems with a Free Final State . . . . .	38
2.3.1	The Optimal Control Problem of Type C . . . . .	38
2.3.2	Pontryagin's Minimum Principle . . . . .	38
2.3.3	Proof . . . . .	39
2.3.4	The LQ Regulator Problem . . . . .	41
2.4	Optimal Control Problems with a Partially Constrained Final State . . . . .	43
2.4.1	The Optimal Control Problem of Type B . . . . .	43
2.4.2	Pontryagin's Minimum Principle . . . . .	43
2.4.3	Proof . . . . .	44
2.4.4	Energy-Optimal Control . . . . .	46
2.5	Optimal Control Problems with State Constraints . . . . .	48
2.5.1	The Optimal Control Problem of Type D . . . . .	48
2.5.2	Pontryagin's Minimum Principle . . . . .	49
2.5.3	Proof . . . . .	51
2.5.4	Time-Optimal, Frictionless, Horizontal Motion of a Mass Point with a Velocity Constraint . . . . .	54
2.6	Singular Optimal Control . . . . .	59
2.6.1	Problem Solving Technique . . . . .	59
2.6.2	Goh's Fishing Problem . . . . .	60
2.6.3	Fuel-Optimal Atmospheric Flight of a Rocket . . . . .	62
2.7	Existence Theorems . . . . .	65
2.8	Optimal Control Problems with a Non-Scalar-Valued Cost Functional . . . . .	67
2.8.1	Introduction . . . . .	67
2.8.2	Problem Statement . . . . .	68
2.8.3	Geering's Infimum Principle . . . . .	68
2.8.4	The Kalman-Bucy Filter . . . . .	69
2.9	Exercises . . . . .	72
<b>3</b>	<b>Optimal State Feedback Control . . . . .</b>	<b>75</b>
3.1	The Principle of Optimality . . . . .	75
3.2	Hamilton-Jacobi-Bellman Theory . . . . .	78
3.2.1	Sufficient Conditions for the Optimality of a Solution . . . . .	78
3.2.2	Plausibility Arguments about the HJB Theory . . . . .	80

3.2.3	The LQ Regulator Problem . . . . .	81
3.2.4	The Time-Invariant Case with Infinite Horizon . . . . .	83
3.3	Approximatively Optimal Control . . . . .	86
3.3.1	Notation . . . . .	87
3.3.2	Lukes' Method . . . . .	88
3.3.3	Controller with a Progressive Characteristic . . . . .	92
3.3.4	LQQ Speed Control . . . . .	96
3.4	Exercises . . . . .	99
<b>4</b>	<b>Differential Games</b> . . . . .	<b>103</b>
4.1	Theory . . . . .	103
4.1.1	Problem Statement . . . . .	104
4.1.2	The Nash-Pontryagin Minimax Principle . . . . .	105
4.1.3	Proof . . . . .	106
4.1.4	Hamilton-Jacobi-Isaacs Theory . . . . .	107
4.2	The LQ Differential Game Problem . . . . .	109
4.2.1	... Solved with the Nash-Pontryagin Minimax Principle	109
4.2.2	... Solved with the Hamilton-Jacobi-Isaacs Theory . .	111
4.3	$H_\infty$ -Control via Differential Games . . . . .	113
	<b>Solutions to Exercises</b> . . . . .	<b>117</b>
	<b>References</b> . . . . .	<b>129</b>
	<b>Index</b> . . . . .	<b>131</b>