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Chapter 14 of volume 1 and Chapter 26 of volume 2 are available in pdf format on the CD-ROM attached to volume 1. The system requirements are Windows 98 and higher. Both pdf files contain links to computer animations. To see the animations, one double clicks on the active links contained inside the pdf documents. The animations are then displayed in an internet browser, such Microsoft Internet Explorer or Netscape. Alternatively, gif-file animations are also provided.

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