
Contents

Introduction	1
<hr/>	
Part I Wigner Matrices and Moments Estimates	5
<hr/>	
1 Wigner's Theorem	7
1.1 Catalan Numbers, Non-crossing Partitions and Dick Paths	7
1.2 Wigner's Theorem	16
1.3 Weak Convergence of the Spectral Measure	20
1.4 Relaxation of the Hypotheses over the Entries–Universality ...	22
2 Wigner's Matrices; More Moments Estimates	29
2.1 Central Limit Theorem	29
2.2 Estimates of the Largest Eigenvalue of Wigner Matrices	33
3 Words in Several Independent Wigner Matrices	41
3.1 Partitions of Colored Elements and Stars	41
3.2 Voiculescu's Theorem	42
<hr/>	
Part II Wigner Matrices and Concentration Inequalities	47
<hr/>	
4 Concentration Inequalities and Logarithmic Sobolev Inequalities	49
4.1 Concentration Inequalities for Laws Satisfying Logarithmic Sobolev Inequalities	49
4.2 A Few Laws Satisfying a Log-Sobolev Inequality	52

5	Generalizations	59
5.1	Concentration Inequalities for Laws Satisfying Weaker Coercive Inequalities	59
5.2	Concentration Inequalities by Talagrand's Method	60
5.3	Concentration Inequalities on Compact Riemannian Manifold with Positive Ricci Curvature	61
5.4	Local Concentration Inequalities	62
6	Concentration Inequalities for Random Matrices	65
6.1	Smoothness and Convexity of the Eigenvalues of a Matrix	65
6.2	Concentration Inequalities for the Eigenvalues of Random Matrices	70
6.3	Concentration Inequalities for Traces of Several Random Matrices	72
6.4	Concentration Inequalities for the Haar Measure on $O(N)$	74
6.5	Brascamp–Lieb Inequalities; Applications to Random Matrices	77
<hr/>		
Part III	Matrix Models	89
<hr/>		
7	Maps and Gaussian Calculus	93
7.1	Combinatorics of Maps and Non-commutative Polynomials	93
7.2	Non-commutative Polynomials	93
7.3	Maps and Polynomials	97
7.4	Formal Expansion of Matrix Integrals	99
8	First-order Expansion	109
8.1	Finite-dimensional Schwinger–Dyson Equations	109
8.2	Tightness and Limiting Schwinger–Dyson Equations	110
8.3	Convergence of the Empirical Distribution	113
8.4	Combinatorial Interpretation of the Limit	114
8.5	Convergence of the Free Energy	118
9	Second-order Expansion for the Free Energy	121
9.1	Rough Estimates on the Size of the Correction $\tilde{\delta}_t^N$	122
9.2	Central Limit Theorem	124
9.3	Comments on the Results	137
9.4	Second-order Correction to the Free Energy	140

Part IV	Eigenvalues of Gaussian Wigner Matrices and Large Deviations	147
----------------	---------------------------------------------------------------------	------------

10	Large Deviations for the Law of the Spectral Measure of Gaussian Wigner’s Matrices	149
11	Large Deviations of the Maximum Eigenvalue	159

Part V	Stochastic Calculus	165
---------------	----------------------------	------------

12	Stochastic Analysis for Random Matrices	167
	12.1 Dyson’s Brownian Motion	167
	12.2 Itô’s Calculus	175
	12.3 A Dynamical Proof of Wigner’s Theorem 1.13	176
13	Large Deviation Principle for the Law of the Spectral Measure of Shifted Wigner Matrices	183
	13.1 Large Deviations from the Hydrodynamical Limit for a System of Independent Brownian Particles	186
	13.2 Large Deviations for the Law of the Spectral Measure of a Non-centered Large Dimensional Matrix-valued Brownian Motion	192
14	Asymptotics of Harish–Chandra–Itzykson–Zuber Integrals and of Schur Polynomials	211
15	Asymptotics of Some Matrix Integrals	217
	15.1 Enumeration of Maps from Matrix Models	220
	15.2 Enumeration of Colored Maps from Matrix Models	222

Part VI	Free Probability	225
----------------	-------------------------	------------

16	Free Probability Setting	227
	16.1 A Few Notions about Algebras and Tracial States	227
	16.2 Space of Laws of m Non-commutative Self-adjoint Variables	228
17	Freeness	231
	17.1 Definition of Freeness	231
	17.2 Asymptotic Freeness	232
	17.3 The Combinatorics of Freeness	236
18	Free Entropy	245

Part VII	Appendix	261
-----------------	-----------------	------------

19	Basics of Matrices	263
	19.1 Weyl's and Lidskii's Inequalities	263
	19.2 Non-commutative Hölder Inequality	264
20	Basics of Probability Theory	267
	20.1 Basic Notions of Large Deviations	267
	20.2 Basics of Stochastic Calculus	270
	20.3 Proof of (2.3)	274
	References	275
	Index	287
	List of Participants of the Summer School	289
	List of Short Lectures Given at the Summer School	293