

# Contents

<b>Introduction</b> .....	1
<b>1 Optimal control</b> .....	11
1.1 Statement of the optimal control problem .....	11
1.2 The maximum principle .....	16
1.3 Open-loop and feedback control .....	21
1.4 Examples .....	23
<b>2 Method of decomposition (the first approach)</b> .....	31
2.1 Problem statement and game approach .....	31
2.1.1 Controlled mechanical system .....	31
2.1.2 Simplifying assumptions .....	32
2.1.3 Decomposition .....	35
2.1.4 Game problem .....	36
2.2 Control of the subsystem and feedback control design .....	37
2.2.1 Optimal control for the subsystem .....	37
2.2.2 Simplified control for the subsystem .....	42
2.2.3 Comparative analysis of the results .....	45
2.2.4 Control for the initial system .....	52
2.3 Weak coupling between degrees of freedom .....	54
2.3.1 Modification of the decomposition method .....	54
2.3.2 Analysis of the controlled motions .....	56
2.3.3 Determination of the parameters .....	59
2.3.4 Case of zero initial velocities .....	61
2.4 Nonlinear damping .....	67
2.4.1 Subsystem with nonlinear damping .....	67
2.4.2 Control for the nonlinear subsystem .....	69
2.4.3 Simplified control for the subsystem and comparative analysis .....	74
2.5 Applications and numerical examples .....	82
2.5.1 Application to robotics .....	82

2.5.2	Feedback control design and modelling of motions for two-link manipulator with direct drives . . . . .	86
2.5.3	Modelling of motions of three-link manipulator . . . . .	92
<b>3</b>	<b>Method of decomposition (the second approach)</b> . . . . .	<b>103</b>
3.1	Problem statement and game approach . . . . .	103
3.1.1	Controlled mechanical system . . . . .	103
3.1.2	Statement of the problem . . . . .	105
3.1.3	Control in the absence of external forces . . . . .	106
3.1.4	Decomposition . . . . .	108
3.2	Feedback control design and its generalizations . . . . .	112
3.2.1	Feedback control design . . . . .	112
3.2.2	Control in the general case . . . . .	114
3.2.3	Extension to the case of nonzero terminal velocity . . . . .	117
3.2.4	Tracking control for mechanical system . . . . .	124
3.3	Applications to robots . . . . .	131
3.3.1	Symbolic generation of equations for multibody systems . . . . .	131
3.3.2	Modelling of control for a two-link mechanism (with three degrees of freedom) . . . . .	136
3.3.3	Modelling of tracking control for a two-link mechanism (with two degrees of freedom) . . . . .	144
<b>4</b>	<b>Stability based control for Lagrangian mechanical systems</b> . . . . .	<b>147</b>
4.1	Scleronomic and rheonomic mechanical systems . . . . .	147
4.2	Lyapunov stability of equilibrium . . . . .	151
4.3	Lyapunov's direct method for autonomous systems . . . . .	151
4.4	Lyapunov's direct method for nonautonomous systems . . . . .	153
4.5	Stabilization of mechanical systems . . . . .	153
4.6	Modification of Lyapunov's direct method . . . . .	155
<b>5</b>	<b>Piecewise linear control for mechanical systems under uncertainty</b> . . . . .	<b>157</b>
5.1	Piecewise linear control for scleronomic systems . . . . .	157
5.1.1	Problem statement . . . . .	157
5.1.2	Description of the control algorithm . . . . .	159
5.1.3	Justification of the algorithm . . . . .	161
5.1.4	Estimation of the time of motion . . . . .	166
5.1.5	Sufficient condition for steering the system to the prescribed state . . . . .	168
5.2	Applications to mechanical systems . . . . .	170
5.2.1	Control of a two-link manipulator . . . . .	170
5.2.2	Control of a two-mass system with unknown parameters . . . . .	173
5.2.3	The first stage of the motion . . . . .	178
5.2.4	The second stage of the motion . . . . .	182
5.2.5	System "a load on a cart" . . . . .	186
5.2.6	System "a pendulum on a cart" . . . . .	187

- 5.2.7 Computer simulation results ..... 197
- 5.3 Piecewise linear control for rheonomic systems ..... 199
  - 5.3.1 Problem statement ..... 199
  - 5.3.2 Control algorithm for rheonomic systems ..... 200
  - 5.3.3 Justification of the control ..... 201
  - 5.3.4 Results of simulation ..... 210
- 6 Continuous feedback control for mechanical systems under uncertainty ..... 213**
  - 6.1 Feedback control for scleronomic system with a given matrix of inertia ..... 213
    - 6.1.1 Problem statement ..... 213
    - 6.1.2 Control function ..... 214
    - 6.1.3 Justification of the control ..... 217
    - 6.1.4 Sufficient condition for controllability ..... 222
    - 6.1.5 Computer simulation results ..... 223
  - 6.2 Control of a scleronomic system with an unknown matrix of inertia 229
    - 6.2.1 Problem statement ..... 229
    - 6.2.2 Computer simulation of the motion of a two-link manipulator 234
  - 6.3 Control of rheonomic systems under uncertainty ..... 237
    - 6.3.1 Problem statement ..... 237
    - 6.3.2 Computer simulation results ..... 241
- 7 Control in distributed-parameter systems ..... 245**
  - 7.1 System of linear oscillators ..... 245
    - 7.1.1 Equations of motion ..... 245
    - 7.1.2 Decomposition ..... 246
    - 7.1.3 Time-optimal control problem ..... 248
    - 7.1.4 Upper bound for the optimal time ..... 249
  - 7.2 Distributed-parameter systems ..... 252
    - 7.2.1 Statement of the control problem for a distributed-parameter system ..... 252
    - 7.2.2 Decomposition ..... 254
    - 7.2.3 First-order equation in time ..... 257
    - 7.2.4 Second-order equation in time ..... 258
    - 7.2.5 Analysis of the constraints and construction of the control .. 259
  - 7.3 Solvability conditions ..... 263
    - 7.3.1 The one-dimensional problems ..... 263
    - 7.3.2 Control of beam oscillations ..... 266
    - 7.3.3 The two-dimensional and three-dimensional problems ..... 267
    - 7.3.4 Solvability conditions in the general case ..... 270

<b>8</b>	<b>Control system under complex constraints</b> .....	275
8.1	Control design in linear systems under complex constraints .....	275
8.1.1	Problem statement .....	275
8.1.2	Kalman's approach .....	277
8.2	Application to oscillating systems .....	281
8.2.1	Control for the system of oscillators .....	281
8.2.2	Pendulum with a suspension point controlled by acceleration .....	286
8.2.3	Pendulum with a suspension point controlled by acceleration (continuation) .....	290
8.2.4	Pendulum with a suspension point controlled by velocity ...	296
8.3	Application to electro-mechanical systems .....	303
8.3.1	Model of the electro-mechanical system .....	303
8.3.2	Analysis of the simplified model .....	306
8.3.3	Control of the electro-mechanical system of the fourth order .....	310
8.3.4	Active dynamical damper .....	317
<b>9</b>	<b>Optimal control problems under complex constraints</b> .....	327
9.1	Time-optimal control problem under mixed and phase constraints ..	328
9.1.1	Problem statement .....	328
9.1.2	Time-optimal control under constraints imposed on the velocity and acceleration .....	329
9.1.3	Problem of control of an electric motor .....	335
9.2	Time-optimal control under constraints imposed on the rate of change of the acceleration .....	340
9.2.1	Statement of the problem .....	340
9.2.2	Open-loop optimal control .....	342
9.2.3	Feedback optimal control .....	345
9.3	Time-optimal control under constraints imposed on the acceleration and its rate .....	354
9.3.1	Problem statement .....	354
9.3.2	Possible modes of control .....	357
9.3.3	Construction of the trajectories .....	359
<b>10</b>	<b>Time-optimal swing-up and damping feedback controls of a nonlinear pendulum</b> .....	367
10.1	Optimal control structure .....	368
10.1.1	Statement of the problem .....	368
10.1.2	Phase cylinder .....	369
10.1.3	Maximum principle .....	370
10.1.4	Numerical algorithm .....	371
10.2	Swing-up control .....	372
10.2.1	Literature overview .....	372
10.2.2	Special trajectories .....	373
10.2.3	Numerical results .....	375
10.3	Damping control .....	380

10.3.1 Literature overview .....	380
10.3.2 Special trajectories .....	382
10.3.3 Numerical results .....	383
References .....	389
<b>Index</b> .....	<b>395</b>

