Contents

Part I: Basics on Synchronization and Paradigmatic Models

1 Introduction 3
- 1.1 Synchronization Phenomena in Nature, Physics, and Engineering 3
- 1.2 Goal of the Book 5
- 1.3 Terminological Remarks 7
- 1.4 Bibliographical Remarks 8

2 Basic Models 11
- 2.1 Harmonic Oscillator: Amplitude, Frequency and Phase of Oscillations 11
- 2.2 Van der Pol Oscillator: Quasi-Harmonic and Relaxation Limit Cycles 12
- 2.3 Rössler Oscillator: From Phase-Coherent to Funnel Chaotic Attractors 14
- 2.4 Lorenz Oscillator: "Classic" and Intermittent Chaotic Attractors 18
- 2.5 Phase Oscillators 21
  - 2.5.1 First-Order Phase Oscillator (Active Rotator) 21
  - 2.5.2 Second-Order Phase Oscillator (Pendulum-Like System) 22
  - 2.5.3 Third-Order Phase Oscillator (Chaotic Rotator) 24
  - 2.5.4 Discrete-Time Rotator (Circle Map) 24
- 2.6 Discrete Map for Spiking–Bursting Neural Activity 28
- 2.7 Excitable Systems 29
  - 2.7.1 Hodgkin–Huxley Model 29
  - 2.7.2 FitzHugh–Nagumo Model 30
  - 2.7.3 Luo–Rudy Model 33

3 Synchronization Due to External Periodic Forcing 35
3.1 Synchronization of Limit-Cycle Oscillator by External Force 36
3.1.1 Weak Forcing: Phase Description 36
3.1.2 Synchronization of a van der Pol Oscillator by External Force 37
3.2 Phase Synchronization of a Chaotic Rössler Oscillator by External Driving 39
3.3 Imperfect Phase Synchronization 42
3.4 Transition to the Regime of Chaotic Phase Synchronization: The Role of Unstable Periodic Orbits 45
3.5 External Phase Synchronization of Chaotic Intermittent Oscillators 47
3.5.1 Forced Model Quadratic Map 47
3.5.2 Forced Lorenz Oscillator 51
3.6 Synchronous Response of Excitable Systems to a Periodic External Force 52
3.7 Conclusions 53

4 Synchronization of Two Coupled Systems 55
4.1 Synchronization of Regular Systems 55
4.1.1 Phase Dynamics Approach 56
4.1.2 Synchronization of Two Coupled van der Pol Oscillators 58
4.1.3 Synchronization of Coupled Active Rotators 66
4.2 Synchronization of Coupled Chaotic Oscillators 68
4.2.1 Phase Synchronization of Rössler Oscillators 68
4.2.2 Synchronization of Coupled Intermittent Oscillators 77
4.2.3 Oscillatory and Rotatory Synchronization of Chaotic Phase Systems 79
4.3 Synchronization of Coupled Circle Maps 90
4.3.1 Regular Synchronization 91
4.3.2 Chaotic Synchronization 93

Part II: Synchronization in Geometrically Regular Ensembles

5 Ensembles of Phase Oscillators 103
5.1 General Model and Malkin's Theorem 104
5.2 Unidirectional Coupling 106
5.3 Synchronization Phenomena in a Chain of Bidirectionally Coupled Phase Oscillators 112
5.3.1 Synchronization, Clustering and Multistability in Chains with Linearly Distributed Individual Frequencies 114

5.3.2 Synchronization Transitions in Chains with Randomly Distributed Individual Frequencies . . . 119
5.4 Influence of Non-Uniform Rotations on the Synchronization . . 121
5.5 Mutual Entrainment in Populations of Globally Coupled Phase Oscillators 123
5.6 Synchronization Phenomena in a Chain of Coupled Pendulum-Like Equations 125
5.7 Conclusions 127

6 Chains of Coupled Limit-Cycle Oscillators 129
6.1 Objectives 130
6.2 Synchronization Clusters and Multistability at Linear Variation of Individual Frequencies Along the Chain 130
6.2.1 Model Equations 131
6.2.2 Global Synchronization in an Assembly, Stationary Phase Distributions, Synchronization area 133
6.2.3 Regimes of Cluster Synchronization 135
6.2.4 Multistability 141
6.3 Oscillation Death 143
6.4 Effects of Nonuniformity of the Frequency Mismatch Gradient in the Formation of Synchronized Clusters 145
6.4.1 Sensitivity of the Structures to Regular Nonuniformities 145
6.4.2 The Effect of Random Dispersion of Individual Frequencies on Cluster Synchronization 146
6.5 Synchronization in a Chain of van der Pol Oscillators 147
6.6 Conclusions 150

7 Ensembles of Chaotic Oscillators with a Periodic-Doubling Route to Chaos, Rössler Oscillators 151
7.1 Synchronization Effects in a Homogeneous Chain of Rössler Oscillators 151
7.2 Basic Model of a Nonhomogeneous Chain, Phase and Frequency Definitions, and Criteria of Phase Synchronization 152
7.3 Phase Synchronization in a Chain with a Linear Distribution of Natural Frequencies, Phase-Coherent Rössler Oscillators . . . 154
7.3.1 Theoretical Study 154
7.3.2 Numerical Results 155
7.4 Synchronization in a Chain with Randomly Distributed Natural Frequencies 160
7.5 Phase Synchronization of Rössler Oscillators with the Funnel Attractor 162

7.6 Anomalous Collective Behavior of Coupled Chaotic Oscillators . . . 165
7.7 Conclusions . . . 167

8 Intermittent-Like Oscillations in Chains of Coupled Maps . . . 169
8.1 Model of Coupled Intermittent Maps, Phase and Frequency, Synchronization Criteria . . . 170
8.2 Linearly Distributed Control Parameters, Soft Transition to Global Synchronization Regime . . . 171
8.3 Randomly Distributed Control Parameter, Transition to Spatiotemporal Intermittency . . . 173
8.4 Collective Oscillations in a Chain of Spiking Maps . . . 177
8.5 Synchronization in Ensembles of Globally Coupled Bursting Oscillators . . . 178
8.5.1 Mutual Synchronization . . . 180
8.5.2 External Synchronization . . . 182
8.6 Conclusions . . . 185

9 Regular and Chaotic Phase Synchronization of Coupled Circle Maps . . . 187
9.1 Common Model for a Chain of Coupled Circle Maps . . . 188
9.2 Synchronization in a Chain of Identical Circle Maps . . . 189
9.2.1 Symmetrically Coupled Maps . . . 190
9.2.2 Effect of Asymmetry of Coupling . . . 195
9.2.3 Synchronization in Lattices of Coupled Maps . . . 197
9.3 Ensembles of Coupled Nonidentical Circle Maps and Criteria of Synchronization . . . 199
9.4 Synchronization and Clustering in a Chain of Regular CMs . . 200
9.4.1 Linear Distribution of Individual Frequencies . . . 200
9.4.2 Random Distribution of Individual Frequencies . . . 206
9.5 Chaotic Phase Synchronization . . . 207
9.6 Conclusions . . . 208

10 Controlling Phase Synchronization in Oscillatory Networks . . . 213
10.1 General Principles of Automatic Synchronization . . . 214
10.2 Two Coupled Poincaré Systems . . . 216
10.3 Coupled van der Pol and Rössler Oscillators . . . 217
10.4 Two Coupled Rössler Oscillators . . . 220
10.5 Coupled Rössler and Lorenz Oscillators . . . 223
10.6 Principles of Automatic Synchronization in Networks of Coupled Oscillators . . . 224
10.7 Synchronization of Locally Coupled Regular Oscillators . . . 225
10.8 Synchronization of Locally Coupled Chaotic Oscillators . . . 228

10.9 Synchronization of Globally Coupled Chaotic Oscillators 230
10.10 Conclusions 231

11 Chains of Limit-Cycle Oscillators 233
11.1 Introduction and Model 233
11.2 Mechanism of Localized Structure Formation 235
11.3 Dissipative Coupling (Zero "Dispersion") 235
11.3.1 Desynchronization of Front Propagation 235
11.3.2 Localized Synchronization Structures 237
11.3.3 Nonlocal Synchronization in Nonhomogeneous Chains 238
11.3.4 Fully Incoherent (Turbulent-Like) Oscillations 239
11.4 Nonscalar (Dissipative and Conservative) Coupling 241
11.4.1 Bursting Structures 241
11.4.2 Nonpropagation to Propagation Transition via Intermittency 242
11.4.3 Noise Influence 247
11.5 Conclusions 248

12 Chains and Lattices of Excitable Luo–Rudy Systems 251
12.1 Objectives 252
12.2 Cardiac Model 253
12.3 Methods: Theoretical Basis 254
12.4 Computational Results 255
12.4.1 One-Dimensional Simulations 255
12.4.2 Two-Dimensional Simulations 261
12.5 Conclusions 265

Part III: Synchronization in Complex Networks and Influence of Noise

13 Noise-Induced Synchronization in Ensembles of Oscillatory and Excitable Systems 269
13.1 Degrading Effects of Noise: Noise-Induced Phase Slips 270
13.2 Noise-Induced CS and PS in Uncoupled Chaotic Oscillators . . 273
13.2.1 Noise-Induced CS of Identical Chaotic Oscillators 273
13.2.2 Noise-Induced PS of Nonidentical Uncoupled Chaotic Systems 285
13.3 Noise-Enhanced PS in Weakly Coupled Chaotic Oscillators . . . 288
13.3.1 Noise-Enhanced PS of a Chaotic Laser Due to Periodic Forcing 289
13.3.2 Noise-Enhanced PS of Two Coupled Rössler Oscillators 292

13.3.3 Noise-Enhanced PS in Arrays of Globally Coupled Rössler Oscillators . . . 295
13.3.4 Experimental Observation of Noise-Enhanced PS 297
13.4 Noise-Enhanced Synchronization-Like Phenomena in Arrays of Coupled Excitable Cells . . . 305
13.4.1 Phase Synchrony in Chains of Coupled Noisy Excitable Neurons . . . 305
13.4.2 Noise-Enhanced PS of Coupled Excitable Neurons by External Forcing . . . 309
13.4.3 Resonant Pattern Formation in 2D Arrays . . . 313
13.5 Conclusions . . . 315

14 Networks with Complex Topology . . . 317
14.1 Introduction . . . 317
14.2 Dynamical Equations and Stability Analysis . . . 320
14.3 Phase Synchronization in Small-World Networks of Oscillators . . . 321
14.4 Synchronization in Scale-Free Networks of Oscillators . . . 324
14.5 Mean-Field Analysis of Hierarchical Synchronization . . . 331
14.6 Synchronization Properties of Weighted Networks . . . 332
14.7 Conclusions . . . 339

Glossary . . . 341

Acknowledgments . . . 343

References . . . 345

Index . . . 363