

# Table of Contents

<b>1</b>	<b>Introduction</b>	1
1.1	Enumeration	2
1.2	Running Time of Algorithms	5
1.3	Linear Optimization Problems	8
1.4	Sorting	9
	Exercises	11
	References	12
<b>2</b>	<b>Graphs</b>	13
2.1	Basic Definitions	13
2.2	Trees, Circuits, and Cuts	17
2.3	Connectivity	24
2.4	Eulerian and Bipartite Graphs	30
2.5	Planarity	33
2.6	Planar Duality	40
	Exercises	43
	References	46
<b>3</b>	<b>Linear Programming</b>	49
3.1	Polyhedra	50
3.2	The Simplex Algorithm	54
3.3	Implementation of the Simplex Algorithm	57
3.4	Duality	60
3.5	Convex Hulls and Polytopes	64
	Exercises	66
	References	68
<b>4</b>	<b>Linear Programming Algorithms</b>	71
4.1	Size of Vertices and Faces	71
4.2	Continued Fractions	74
4.3	Gaussian Elimination	77
4.4	The Ellipsoid Method	80
4.5	Khachiyan's Theorem	86
4.6	Separation and Optimization	88
	Exercises	95
	References	96

<b>5</b>	<b>Integer Programming</b> .....	99
	5.1 The Integer Hull of a Polyhedron .....	101
	5.2 Unimodular Transformations .....	105
	5.3 Total Dual Integrality .....	107
	5.4 Totally Unimodular Matrices .....	110
	5.5 Cutting Planes .....	115
	5.6 Lagrangean Relaxation .....	119
	Exercises .....	121
	References .....	125
<b>6</b>	<b>Spanning Trees and Arborescences</b> .....	127
	6.1 Minimum Spanning Trees .....	127
	6.2 Minimum Weight Arborescences .....	133
	6.3 Polyhedral Descriptions .....	137
	6.4 Packing Spanning Trees and Arborescences .....	140
	Exercises .....	144
	References .....	147
<b>7</b>	<b>Shortest Paths</b> .....	151
	7.1 Shortest Paths From One Source .....	152
	7.2 Shortest Paths Between All Pairs of Vertices .....	156
	7.3 Minimum Mean Cycles .....	159
	Exercises .....	161
	References .....	163
<b>8</b>	<b>Network Flows</b> .....	165
	8.1 Max-Flow-Min-Cut Theorem .....	166
	8.2 Menger's Theorem .....	170
	8.3 The Edmonds-Karp Algorithm .....	172
	8.4 Blocking Flows and Fujishige's Algorithm .....	174
	8.5 The Goldberg-Tarjan Algorithm .....	176
	8.6 Gomory-Hu Trees .....	180
	8.7 The Minimum Capacity of a Cut in an Undirected Graph .....	186
	Exercises .....	189
	References .....	194
<b>9</b>	<b>Minimum Cost Flows</b> .....	199
	9.1 Problem Formulation .....	199
	9.2 An Optimality Criterion .....	201
	9.3 Minimum Mean Cycle-Cancelling Algorithm .....	203
	9.4 Successive Shortest Path Algorithm .....	207
	9.5 Orlin's Algorithm .....	211
	9.6 The Network Simplex Algorithm .....	214
	9.7 Flows Over Time .....	218
	Exercises .....	220
	References .....	224

<b>10</b>	<b>Maximum Matchings</b>	227
	10.1 Bipartite Matching	228
	10.2 The Tutte Matrix	230
	10.3 Tutte's Theorem	232
	10.4 Ear-Decompositions of Factor-Critical Graphs	235
	10.5 Edmonds' Matching Algorithm	241
	Exercises	250
	References	254
<b>11</b>	<b>Weighted Matching</b>	257
	11.1 The Assignment Problem	258
	11.2 Outline of the Weighted Matching Algorithm	259
	11.3 Implementation of the Weighted Matching Algorithm	262
	11.4 Postoptimality	276
	11.5 The Matching Polytope	277
	Exercises	280
	References	282
<b>12</b>	<b><i>b</i>-Matchings and <i>T</i>-Joins</b>	285
	12.1 <i>b</i> -Matchings	285
	12.2 Minimum Weight <i>T</i> -Joins	289
	12.3 <i>T</i> -Joins and <i>T</i> -Cuts	293
	12.4 The Padberg-Rao Theorem	296
	Exercises	299
	References	302
<b>13</b>	<b>Matroids</b>	305
	13.1 Independence Systems and Matroids	305
	13.2 Other Matroid Axioms	309
	13.3 Duality	313
	13.4 The Greedy Algorithm	317
	13.5 Matroid Intersection	322
	13.6 Matroid Partitioning	327
	13.7 Weighted Matroid Intersection	329
	Exercises	332
	References	334
<b>14</b>	<b>Generalizations of Matroids</b>	337
	14.1 Greedoids	337
	14.2 Polymatroids	341
	14.3 Minimizing Submodular Functions	345
	14.4 Schrijver's Algorithm	347
	14.5 Symmetric Submodular Functions	351
	Exercises	353
	References	355

<b>15</b>	<b><i>NP</i>-Completeness</b>	359
	15.1 Turing Machines	359
	15.2 Church's Thesis	362
	15.3 <i>P</i> and <i>NP</i>	367
	15.4 Cook's Theorem	371
	15.5 Some Basic <i>NP</i> -Complete Problems	375
	15.6 The Class <i>coNP</i>	382
	15.7 <i>NP</i> -Hard Problems	384
	Exercises	387
	References	391
<b>16</b>	<b>Approximation Algorithms</b>	393
	16.1 Set Covering	394
	16.2 The Max-Cut Problem	399
	16.3 Colouring	405
	16.4 Approximation Schemes	413
	16.5 Maximum Satisfiability	415
	16.6 The <i>PCP</i> Theorem	420
	16.7 L-Reductions	424
	Exercises	430
	References	434
<b>17</b>	<b>The Knapsack Problem</b>	439
	17.1 Fractional Knapsack and Weighted Median Problem	439
	17.2 A Pseudopolynomial Algorithm	442
	17.3 A Fully Polynomial Approximation Scheme	444
	Exercises	447
	References	447
<b>18</b>	<b>Bin-Packing</b>	449
	18.1 Greedy Heuristics	449
	18.2 An Asymptotic Approximation Scheme	455
	18.3 The Karmarkar-Karp Algorithm	459
	Exercises	462
	References	464
<b>19</b>	<b>Multicommodity Flows and Edge-Disjoint Paths</b>	467
	19.1 Multicommodity Flows	468
	19.2 Algorithms for Multicommodity Flows	471
	19.3 Directed Edge-Disjoint Paths Problem	476
	19.4 Undirected Edge-Disjoint Paths Problem	479
	Exercises	485
	References	487

<b>20</b>	<b>Network Design Problems</b>	491
	20.1 Steiner Trees	492
	20.2 The Robins-Zelikovsky Algorithm	497
	20.3 Survivable Network Design	502
	20.4 A Primal-Dual Approximation Algorithm	505
	20.5 Jain's Algorithm	514
	Exercises	520
	References	522
<b>21</b>	<b>The Traveling Salesman Problem</b>	527
	21.1 Approximation Algorithms for the TSP	527
	21.2 Euclidean TSP	532
	21.3 Local Search	539
	21.4 The Traveling Salesman Polytope	545
	21.5 Lower Bounds	551
	21.6 Branch-and-Bound	553
	Exercises	556
	References	559
<b>22</b>	<b>Facility Location</b>	563
	22.1 The Uncapacitated Facility Location Problem	563
	22.2 Rounding Linear Programming Solutions	565
	22.3 Primal-Dual Algorithms	567
	22.4 Scaling and Greedy Augmentation	573
	22.5 Bounding the Number of Facilities	576
	22.6 Local Search	579
	22.7 Capacitated Facility Location Problems	585
	22.8 Universal Facility Location	588
	Exercises	594
	References	596
	<b>Notation Index</b>	599
	<b>Author Index</b>	603
	<b>Subject Index</b>	613