

Contents

1	Graph Theory and Small-World Networks	1
1.1	Random Graphs	1
1.1.1	The Small-World Effect	1
1.1.2	Basic Graph-Theoretical Concepts	3
1.1.3	Properties of Random Graphs	7
1.2	Generalized Random Graphs	12
1.2.1	Graphs with Arbitrary Degree Distributions	13
1.2.2	Probability Generating Function Formalism	17
1.2.3	Distribution of Component Sizes	19
1.3	Robustness of Random Networks	22
1.4	Small-World Models	26
1.5	Scale-Free Graphs	28
	Exercises	32
	Further Reading	33
2	Chaos, Bifurcations and Diffusion	35
2.1	Basic Concepts of Dynamical Systems Theory	35
2.2	The Logistic Map and Deterministic Chaos	40
2.3	Dissipation and Adaption	45
2.3.1	Dissipative Systems and Strange Attractors	45
2.3.2	Adaptive Systems	50
2.4	Diffusion and Transport	54
2.4.1	Random Walks, Diffusion and Lévy Flights	54
2.4.2	The Langevin Equation and Diffusion	58
2.5	Noise-Controlled Dynamics	60
2.5.1	Stochastic Escape	60
2.5.2	Stochastic Resonance	63
2.6	Dynamical Systems with Time Delays	66
	Exercises	69
	Further Reading	70

3 Random Boolean Networks	73
3.1 Introduction	73
3.2 Random Variables and Networks	75
3.2.1 Boolean Variables and Graph Topologies	75
3.2.2 Coupling Functions	77
3.2.3 Dynamics	79
3.3 The Dynamics of Boolean Networks	80
3.3.1 The Flow of Information Through the Network	80
3.3.2 The Mean-Field Phase Diagram	83
3.3.3 The Bifurcation Phase Diagram	84
3.3.4 Scale-Free Boolean Networks	88
3.4 Cycles and Attractors	90
3.4.1 Quenched Boolean Dynamics	90
3.4.2 The $K = 1$ Kauffman Network	93
3.4.3 The $K = 2$ Kauffman Network	94
3.4.4 The $K = N$ Kauffman Network	94
3.5 Applications	97
3.5.1 Living at the Edge of Chaos	97
3.5.2 The Yeast Cell Cycle	98
3.5.3 Application to Neural Networks	101
Exercises	102
Further Reading	103
4 Cellular Automata and Self-Organized Criticality	105
4.1 The Landau Theory of Phase Transitions	105
4.2 Criticality in Dynamical Systems	110
4.2.1 $1/f$ Noise	114
4.3 Cellular Automata	115
4.3.1 Conway's Game of Life	116
4.3.2 The Forest Fire Model	117
4.4 The Sandpile Model and Self-Organized Criticality	118
4.5 Random Branching Theory	121
4.6 Application to Long-Term Evolution	125
Exercises	132
Further Reading	133
5 Statistical Modeling of Darwinian Evolution	135
5.1 Introduction	135
5.2 Mutations and Fitness in a Static Environment	137
5.3 Deterministic Evolution	140
5.3.1 Evolution Equations	141
5.3.2 Beanbag Genetics – Evolutions Without Epistasis	144
5.3.3 Epistatic Interactions and the Error Catastrophe	146
5.4 Finite Populations and Stochastic Escape	150
5.4.1 Strong Selective Pressure and Adaptive Climbing	151
5.4.2 Adaptive Climbing Versus Stochastic Escape	154

5.5	Prebiotic Evolution	155
5.5.1	Quasispecies Theory	156
5.5.2	Hypercycles and Autocatalytic Networks	157
5.6	Coevolution and Game Theory	160
	Exercises	165
	Further Reading	166
6	Synchronization Phenomena	169
6.1	Frequency Locking	169
6.2	Synchronization of Coupled Oscillators	170
6.3	Synchronization with Time Delays	176
6.4	Synchronization of Relaxation Oscillators	178
6.5	Synchronization and Object Recognition in Neural Networks	182
6.6	Synchronization Phenomena in Epidemics	185
	Exercises	188
	Further Reading	189
7	Elements of Cognitive Systems Theory	191
7.1	Introduction	191
7.2	Foundations of Cognitive Systems Theory	193
7.2.1	Basic Requirements for the Dynamics	193
7.2.2	Cognitive Information Processing Versus Diffusive Control	197
7.2.3	Basic Layout Principles	199
7.2.4	Learning and Memory Representations	201
7.3	Motivation, Benchmarks and Diffusive Emotional Control	205
7.3.1	Cognitive Tasks	205
7.3.2	Internal Benchmarks	206
7.4	Competitive Dynamics and Winning Coalitions	209
7.4.1	General Considerations	210
7.4.2	Associative Thought Processes	214
7.4.3	Autonomous Online Learning	218
7.5	Environmental Model Building	220
7.5.1	The Elman Simple Recurrent Network	221
7.5.2	Universal Prediction Tasks	224
	Exercises	227
	Further Reading	228
8	Solutions	231
Index	253