

Contents

Preface	vii
---------------	-----

1 Introduction	1
----------------------	---

Part I General Mechanisms of Synchronization

2 General Remarks	9
2.1 What Are We Going to Talk About?	9
2.2 Topics to Consider	10
2.3 Self-Sustained Oscillations: A Key Concept in Synchronization	
Theory	12
2.3.1 Features of Self-Oscillations	12
2.3.2 Features of Self-Oscillating Systems	13
2.3.3 Modern Revisions of the Definition of a Self-Sustained System	16
2.3.4 Self-Sustained Oscillations and Attractors	17
2.3.5 Synchronization as a Control Tool	17
2.4 Duality of the Description of Synchronization	17
2.5 Oscillations Helping Each Other Out	18
2.6 Terms of Bifurcations Theory	19
3 1:1 Forced Synchronization of Periodic Oscillations	21
3.1 Phase of Quasiharmonic Oscillations	24
3.2 Derivation of Truncated Equations for Phase Difference and Amplitude	26
3.3 Amplitude of Unperturbed Oscillations at Small Non-linearity	31
3.4 Analysis of Truncated Equations for Weak Forcing	32
3.5 Derivation of Truncated Equations in Descartes Coordinates	34
3.6 Analysis of Truncated Equations in Descartes Coordinates	37
3.7 Synchronization Region from the Truncated Equations: Non-bifurcational Approach	45
3.8 Fourier Power Spectra at Strong Forcing	50
3.9 Phase Locking and Suppression: Numerical Simulation	56

3.9.1	Phase Locking	56
3.9.2	Suppression of Natural Dynamics	60
3.10	Phase Locking and Suppression: Experiment	62
3.10.1	Amplitudes from Oscilloscopes	64
3.11	Beat Frequency: Theory, Simulations and Experiment	67
3.11.1	Theory	67
3.11.2	Numerical Simulation	71
3.11.3	Experiment	72
4	1:1 Mutual Synchronization of Periodic Oscillations	75
4.1	Truncated Equations for Weakly Non-linear Oscillators	77
4.2	Periodic Oscillators with Dissipative Coupling	80
4.2.1	Symmetric Solutions	81
4.2.2	Asymmetric Solutions	83
4.2.3	Oscillation Death	84
4.3	Dissipative Coupling: Numerical Simulation	84
4.3.1	Locking	85
4.3.2	Bifurcations	86
4.3.3	Suppression	87
4.4	Reactive Coupling	89
4.4.1	Locking	90
4.4.2	Suppression	92
4.4.3	Bifurcations	92
4.4.4	Phase Multistability	94
4.5	Reactive Coupling and the Saddle Torus	95
4.5.1	Hypothesized Structure of the Phase Space	96
4.6	Generality of Bifurcational Transitions at Reactive Coupling	97
4.7	Experiment	99
4.7.1	Phase Locking	100
4.7.2	Suppression	101
4.8	Comparison of Synchronization Transitions in Forced and in Mutually Coupled Oscillators	103
5	Homoclinic Mechanism of Synchronization of Periodic Oscillations	105
5.1	Global Bifurcation	108
5.1.1	Features of a Homoclinic Bifurcation of a Cycle	110
5.2	Homoclinics Inside Synchronization Tongue?	111
5.3	How Homoclinics Leads to Synchronization	114
5.4	Synchronization in a Bacteria–Viruses Model	117
5.5	Summary	120
6	$n:m$ Synchronization of Periodic Oscillations	121
6.1	Important Definitions Relevant to $n:m$ Synchronization	121
6.1.1	Poincaré Return Time	121
6.1.2	Phase of Oscillations	122

6.1.3	Phase of Oscillations via Poincaré Section	122
6.1.4	Poincaré Winding (Rotation) Number	123
6.1.5	Synchronization Order $n:m$	123
6.2	1:1 Forced Synchronization in Weakly Non-linear Oscillators	123
6.2.1	3:1 Phase (Frequency) Locking	128
6.2.2	3:1 Suppression of Natural Dynamics	131
6.3	$n:m$ Synchronization in Strongly Non-linear Oscillators with Spiky Forcing	133
6.3.1	2:3 Phase (Frequency) Locking	136
6.3.2	The Route to 2:3 Suppression	138
6.4	Circle Map: Derivation	138
6.4.1	Amplitude and Phase of Oscillations	139
6.4.2	From Differential to Discrete Equation for Phase	141
6.5	Circle Map: Properties	142
6.6	Arnold Tongues	144
6.7	$n:m$ Synchronization: Experiment	144
6.8	Summary	147
7	1:1 Forced Synchronization of Periodic Oscillations in the Presence of Noise	149
7.1	Introductory Comments on Random Processes	150
7.1.1	One-Dimensional Probability Density, Mean and Variance	150
7.1.2	Two-Dimensional Probability Density, Correlation and Covariance	152
7.1.3	Stationary Process	154
7.1.4	Correlation Time	154
7.1.5	Correlation Between Two Different Processes	155
7.1.6	Spectrum of a Wide-Sense Stationary Process	156
7.2	Truncated Equations	158
7.3	Simplification of the Fluctuational Terms in Truncated Equations	158
7.4	Probability Density Distribution of the Phase Difference	165
7.4.1	Case of $Q > 0$	169
7.5	Bessel Functions	170
7.6	Probability Density Distribution of the Phase Difference, Continued	172
7.7	Mean Frequency of Forced Oscillations with Noise	174
7.8	Interpretation of Phase Dynamics	177
7.9	Phase Diffusion	180
7.10	Full-Scale Biological Experiment	183
7.11	Effects of Noise on the Spectrum of a Synchronized System	185
7.11.1	Effect of Noise on the Spectrum of Oscillations Synchronized by Suppression	189

8	Chaos Synchronization	191
8.1	What Is Chaos?	192
8.1.1	Exponential Divergence of Phase Trajectories	192
8.1.2	Chaos Properties in Terms of Phase Space	193
8.1.3	Chaos Properties in Terms of Spectra	197
8.2	What Does Synchronization of Chaos Encompass?	197
8.2.1	Chaos Synchronization: Different Manifestations	197
8.2.2	Chaos Synchronization in a Classical Sense	198
8.3	Phase and Basic Frequency of Chaotic Oscillations	199
8.4	Forcing Chaos Periodically: What to Expect?	201
8.4.1	Phase Locking of Chaos	203
8.4.2	Suppression of Chaos	204
8.4.3	Any Other Options?	204
8.4.4	Interacting Chaotic Systems	205
8.5	Synchronization of Chaos by Periodic Forcing	205
8.5.1	Experiment	205
8.5.2	Numerical Analysis	211
8.6	Synchronization of Periodic Oscillations by Chaos	212
8.6.1	Spectra	213
8.6.2	Poincaré Sections	215
8.6.3	Phase Difference	216
8.6.4	Lyapunov Exponents	220
8.7	Mutual Synchronization of Chaos	222
8.7.1	Phase/Frequency Locking	222
8.7.2	Suppression	223
8.7.3	Phase Behavior	225
8.8	Homoclinic Synchronization of Chaos	227
8.9	Effects of Noise on a Synchronized Chaos	232
8.9.1	Chaotic System Frequency-Locked by a Harmonic Signal	233
8.9.2	Periodic System Suppressed by Chaotic Forcing	237
8.10	Summary	237
9	Synchronization of Noise-Induced Oscillations	239
	Stochastic Limit Cycle	241
9.1	Noise-Induced Oscillations	242
9.2	Models	243
9.2.1	Morris–Lecar Model	243
9.2.2	Monovibrator Circuit	244
9.3	Coherence Resonance Oscillator	244
9.4	Frequency and Phase Locking	248
9.4.1	Frequency Locking: Electronic Experiment	249
9.4.2	Phase Locking: Coupled Morris–Lecar Models	251
9.4.3	Phase Dynamics Inside the Synchronization Region: Electronic Experiment	253
9.5	Synchronization via Suppression	255

10 Conclusions to Part I	259
---------------------------------------	------------

Part II Case Studies in Synchronization

11 Synchronization of Anisochronous Oscillators	265
11.1 Phase Velocity Field and Coupling Vector	266
11.2 Effective Coupling Function	268
11.2.1 Asymptotic Phase	268
11.2.2 Effective Coupling Function	269
11.3 Dephasing	270
11.4 Examples of 2D Anisochronous Oscillators	273
11.5 Synchronization near the Homoclinic Bifurcation	279
11.5.1 Weak Coupling Limit	282
11.5.2 Finite Coupling Strength	285
11.5.3 Strong Coupling with Moderate μ	288
11.5.4 Summary on Synchronization near Homoclinic Bifurcation	289
11.6 Phase Locking Patterns of Coupled Fast-and-Slow Oscillators	290
11.6.1 Antiphase Locking in Coupled FitzHugh–Nagumo Models	290
11.6.2 Out-of-phase Synchronization via Slow Channels	293
11.7 Synchronous Patterns in Coupled Morris–Lecar Models	296
11.7.1 Model	296
11.7.2 Overview of the Dynamics	298
11.7.3 Structure of Arnold Tongue for Antiphase Solution	300
Chaotic Bursting and Torus Breakdown	306
11.7.4 Crises at the Boundary of Quasiperiodic Regions	308
11.7.5 Transition to In-phase Synchronization	311
11.7.6 Mechanism of Torus Folding in the Vicinity of Unstable Orbit	312
11.7.7 Remarks on Synchronization in Morris–Lecar Systems	314
11.8 Summary	314
12 Phase Multistability	317
12.1 Period-Doubling Oscillations	318
12.1.1 Dynamics of Coupled Rössler Systems	320
12.1.2 Mapping Approach to Multistability	330
12.2 Self-Modulated Oscillations	335
12.2.1 Methods of Analysis	335
12.2.2 Phase Dynamics of Coupled Oscillators	337
12.3 Bursting Dynamics	339
12.3.1 Simple Qualitative Approach to Phase Multistability	342
12.3.2 Dynamics of Coupled Bursters	344
12.3.3 Multistability Induced by Dephasing	349
12.4 Summary	352

13 Synchronization in Systems with Complex Multimode Dynamics	353
13.1 Synchronization of Chaotic Systems with Fast and Slow Time Scales	355
13.1.1 Single System with Two Time Scales	355
13.1.2 Coupled Systems with Two Mode Dynamics	360
13.1.3 Conclusions	363
13.2 Generation and Synchronization of Oscillations with Several Noise-Induced Modes	363
13.2.1 Description of Experiment	364
13.2.2 Characterizing Collective Response by Spectra	364
13.2.3 Mutually Coupled Excitable Units	365
13.2.4 Three Coupled Excitable Units	369
13.2.5 Two Mutually Coupled Excitable Units with Inhibitory Coupling	369
13.3 Synchronization of Chaotic Systems with Denumerable Set of Equilibrium States	371
13.4 Summary	376
14 Synchronization of Systems with Resource Mediated Coupling	377
14.1 Neural Synchronization via Potassium Signaling	379
14.1.1 Model	380
14.1.2 Identical Cells: Competing In-phase and Antiphase Synchronization	383
14.1.3 Heterogeneous Cells: Dynamical Patterns	386
14.2 Multimode Dynamics in Linear Array of Electronic Oscillators	388
14.2.1 Model	388
14.2.2 Clustering	390
14.2.3 Intracluster Synchronization	393
14.3 Cascaded Microbiological Oscillators	395
14.3.1 Model	396
14.3.2 Spatial Dynamics	397
14.4 Synchronization Patterns in Kidney Autoregulation	401
14.4.1 Vascular-Nephron Model	402
14.4.2 Coupling-Induced Inhomogeneity	405
14.5 Summary	408
15 Conclusions to Part II	411
And finally...	412
References	413
Index	423