Contents

1 Overview and Basic Concepts 1
1.1 Introduction 1
1.2 The Programs 5
1.3 Literature on Chaotic Dynamics 8

2 Nonlinear Dynamics and Deterministic Chaos 11
2.1 Deterministic Chaos 12
2.2 Hamiltonian Systems 13
2.2.1 Integrable and Ergodic Systems 13
2.2.2 Poincaré Sections 16
2.2.3 The KAM Theorem 18
2.2.4 Homoclinic Points 20
2.3 Dissipative Dynamical Systems 22
2.3.1 Attractors 24
2.3.2 Routes to Chaos 26
2.4 Special Topics 27
2.4.1 The Poincaré-Birkhoff Theorem 28
2.4.2 Continued Fractions 29
2.4.3 The Lyapunov Exponent 32
2.4.4 Fixed Points of One-Dimensional Maps 35
2.4.5 Fixed Points of Two-Dimensional Maps 38
2.4.6 Bifurcations 44
References 45

3 Billiard Systems 47
3.1 Deformations of a Circle Billiard 50
3.2 Numerical Techniques 53
3.3 Interacting with the Program 54
3.4 Computer Experiments 58
3.4.1 From Regularity to Chaos 58
3.4.2 Zooming In 60

3.4.3 Sensitivity and Determinism 61
3.4.4 Suggestions for Additional Experiments 63
3.5 Suggestions for Further Studies 66
3.6 Real Experiments and Empirical Evidence 66
References 67

4 Gravitational Billiards: The Wedge 69
4.1 The Poincaré Mapping 70
4.2 Interacting with the Program 75
4.3 Computer Experiments 77
4.3.1 Periodic Motion and Phase Space Organization 77
4.3.2 Bifurcation Phenomena 81
4.3.3 'Plane Filling' Wedge Billiards 86
4.3.4 Suggestions for Additional Experiments 88
4.4 Suggestions for Further Studies 89
4.5 Real Experiments and Empirical Evidence 90
References 90

5 The Double Pendulum 91
5.1 Equations of Motion 91
5.2 Numerical Algorithms 93
5.3 Interacting with the Program 93
5.4 Computer Experiments 98
5.4.1 Different Types of Motion 98
5.4.2 Dynamics of the Double Pendulum 102
5.4.3 Destruction of Invariant Curves 107
5.4.4 Suggestions for Additional Experiments 110
5.5 Real Experiments and Empirical Evidence 111
References 113

6 Chaotic Scattering 115
6.1 Scattering off Three Disks 117
6.2 Numerical Techniques 121
6.3 Interacting with the Program 121
6.4 Computer Experiments 124
6.4.1 Scattering Functions and Two-Disk Collisions 124
6.4.2 Tree Organization of Three-Disk Collisions 127
6.4.3 Unstable Periodic Orbits 129
6.4.4 Fractal Singularity Structure 131
6.4.5 Suggestions for Additional Experiments 133
6.5 Suggestions for Further Studies 135
6.6 Real Experiments and Empirical Evidence 136
References 136

7 Fermi Acceleration 137
7.1 Fermi Mapping 138
7.2 Interacting with the Program 139
7.3 Computer Experiments 142
7.3.1 Exploring Phase Space for Different Wall Oscillations 142
7.3.2 KAM Curves and Stochastic Acceleration 144
7.3.3 Fixed Points and Linear Stability 146
7.3.4 Absolute Barriers 148
7.3.5 Suggestions for Additional Experiments 150
7.4 Suggestions for Further Studies 154
7.5 Real Experiments and Empirical Evidence 154
References 155

8 The Duffing Oscillator 157
8.1 The Duffing Equation 157
8.2 Numerical Techniques 161
8.3 Interacting with the Program 161
8.4 Computer Experiments 168
8.4.1 Chaotic and Regular Oscillations 168
8.4.2 The Free Duffing Oscillator 168
8.4.3 Anharmonic Vibrations: Resonances and Bistability 171
8.4.4 Coexisting Limit Cycles and Strange Attractors 174
8.4.5 Suggestions for Additional Experiments 176
8.5 Suggestions for Further Studies 181
8.6 Real Experiments and Empirical Evidence 181
References 183

9 Feigenbaum Scenario 185
9.1 One-Dimensional Maps 186
9.2 Interacting with the Program 188
9.3 Computer Experiments 191
9.3.1 Period-Doubling Bifurcations 191
9.3.2 The Chaotic Regime 195
9.3.3 Lyapunov Exponents 199
9.3.4 The Tent Map 200
9.3.5 Suggestions for Additional Experiments 202
9.4 Suggestions for Further Studies 206
9.5 Real Experiments and Empirical Evidence 208
References 209

10 Nonlinear Electronic Circuits 211
10.1 A Chaos Generator 211
10.2 Numerical Techniques 214
10.3 Interacting with the Program 215
10.4 Computer Experiments 220

10.4.1 Hopf Bifurcation . . . 220
10.4.2 Period-Doubling . . . 221
10.4.3 Return Map . . . 225
10.4.4 Suggestions for Additional Experiments . . . 226
10.5 Real Experiments and Empirical Evidence . . . 229
References . . . 230

11 Mandelbrot and Julia Sets . . . 231
11.1 Two-Dimensional Iterated Maps . . . 231
11.2 Numerical Methods . . . 235
11.3 Interacting with the Program . . . 236
11.4 Computer Experiments . . . 242
11.4.1 Mandelbrot and Julia-sets . . . 242
11.4.2 Zooming into the Mandelbrot Set . . . 244
11.4.3 General Two-Dimensional Quadratic Mappings . . . 245
11.4.4 Suggestions for Additional Experiments . . . 249
11.5 Suggestions for Further Studies . . . 251
11.6 Real Experiments and Empirical Evidence . . . 252
References . . . 253

12 Ordinary Differential Equations . . . 255
12.1 Numerical Techniques . . . 256
12.2 Interacting with the Program . . . 256
12.3 Computer Experiments . . . 268
12.3.1 The Pendulum . . . 268
12.3.2 A Simple Hopf Bifurcation . . . 270
12.3.3 The Duffing Oscillator Revisited . . . 273
12.3.4 Hill's Equation . . . 275
12.3.5 The Lorenz Attractor . . . 281
12.3.6 The Rössler Attractor . . . 284
12.3.7 The Hénon-Heiles System . . . 285
12.3.8 Suggestions for Additional Experiments . . . 288
12.4 Suggestions for Further Studies . . . 293
References . . . 298

13 Kicked Systems . . . 301
13.1 Interacting with the Program . . . 303
13.2 Computer Experiments . . . 307
13.2.1 The Standard Mapping . . . 307
13.2.2 The Kicked Quartic Oscillator . . . 309
13.2.3 The Kicked Quartic Oscillator with Damping . . . 311
13.2.4 The Hénon Map . . . 312
13.2.5 Suggestions for Additional Experiments . . . 313
13.3 Real Experiments and Empirical Evidence . . . 316
References . . . 316

A System Requirements and Program Installation 319
A.1 System Requirements 319
A.2 Installing the Programs 319
A.2.1 Windows Operating System 320
A.2.2 Linux Operating System 320
A.3 Programs 321
A.4 Third Party Software 321

B General Remarks on Using the Programs 323
B.1 Interaction with the Programs 323
B.2 Input of Mathematical Expressions 325

Glossary 327

Index 335