
Contents

1	Overview	1
1.1	Introduction	1
1.2	Microeconomic Foundations	2
1.3	Efficiency Measurement	6
1.4	Productivity and Performance Measurement	8
1.5	Engineering Models of Technology	10
1.6	Mathematical Appendix	13
1.7	A Word of Advice	14

Part I Microeconomic Foundations

2	Production Functions	19
2.1	Parametric Forms	19
2.2	Rate of Technical Substitution	21
2.3	Elasticity	24
2.4	Elasticity of Output, Scale and Returns to Scale	25
2.5	Elasticity of Substitution	26
2.6	Homothetic Production Functions	28
2.7	Exercises	30
2.8	Bibliographical Notes	31
2.9	Solutions to Exercises	32
3	Formal Description of Technology	35
3.1	Primitive Elements	35
3.2	Input and Output Disposability	37
3.3	Efficient Frontiers	38
3.4	Axioms for a Well-Behaved Technology	38
3.5	Single-Output Technologies	39
3.6	Extrapolation of Technology	41
3.6.1	Convexity	41

3.6.2	Disposability	42
3.6.3	Constant Returns-to-Scale	43
3.6.4	Example	45
3.7	Exercises	47
3.8	Bibliographical Notes	48
3.9	Solutions to Exercises	49
4	Nonparametric Models of Technology	53
4.1	Simple Leontief or Fixed-Coefficients Technology	53
4.2	General Leontief Technology	55
4.2.1	Production Function	56
4.2.2	Properties	56
4.2.3	Graphical Construction	58
4.3	Nonparametric Constructions	60
4.3.1	The Hanoch-Rothschild Model of Technology	60
4.3.2	Data Envelopment Analysis Models of Technology	61
4.3.3	Graphical Constructions	63
4.4	Exercises	66
4.5	Bibliographical Notes	67
4.6	Solutions to Exercises	68
5	Cost Function	71
5.1	Definition	71
5.2	Properties	71
5.2.1	Geometry	71
5.2.2	Homogeneity	73
5.2.3	Concavity	73
5.3	Example: Cobb-Douglas Technology	73
5.4	Sensitivity Analysis	76
5.4.1	Sensitivity to Output	76
5.4.2	Sensitivity to Price: Shephard's Lemma	77
5.5	Nonparametric Estimation	78
5.5.1	Leontief Technologies	78
5.5.2	<i>HR</i> Technology	79
5.5.3	<i>CRS</i> and <i>VRS</i> Technologies	79
5.6	Reconstructing the Technology	80
5.6.1	Outer Approximation of Technology	82
5.6.2	Cost and Production	83
5.7	Homothetic Technologies	84
5.8	Appendix	85
5.9	Exercises	86
5.10	Bibliographical Notes	89
5.11	Solutions to Exercises	90

6	Indirect Production Function	97
6.1	Definition	97
6.2	Properties	98
6.3	Duality between the Cost and Indirect Production Functions .	99
6.4	Reconstructing the Technology	100
6.5	Revealed Preference	101
6.6	Nonparametric Estimation	102
6.7	Exercises	103
6.8	Bibliographical Notes.....	104
6.9	Solutions to Exercises	105
7	Distance Functions	109
7.1	Definition	109
	7.1.1 Input Distance Function	109
	7.1.2 Output Distance Function	110
7.2	Properties	111
7.3	Efficiency and Cost	112
7.4	Reconstructing the Input Distance Function from the Cost Function	114
7.5	Application to Homothetic Technologies	117
7.6	Appendix	118
7.7	Exercises	120
7.8	Bibliographical Notes.....	120
7.9	Solutions to Exercises	121
8	Nonconvex Models of Technology	125
8.1	Resource Allocation	125
	8.1.1 Aggregate Production Function	126
	8.1.2 Counter-Example to Quasiconcavity.....	127
8.2	Producer Budgeting	129
	8.2.1 Multi-Dimensional Indirect Production Function.....	129
	8.2.2 Counter-Example to Quasiconvexity.....	129
8.3	Data Envelopment Analysis with Lower Bounds	130
	8.3.1 Fixed-Charge Technology	130
	8.3.2 Nonconvex Geometry of the Fixed-Charge Technology .	132
	8.3.3 The Low Intensity Phenomenon	133
8.4	Projective-Convexity	135
	8.4.1 Definitions and characterizations.....	136
	8.4.2 Separation Properties	139
	8.4.3 Dual Characterization	141
8.5	Exercises	143
8.6	Bibliographical Notes.....	143
8.7	Solutions to Exercises	144

Part II Efficiency Measurement

9 Efficiency Analysis 149

9.1 Input and Output Efficiency 149

9.2 Scale Efficiency 151

9.3 Cost Efficiency 152

9.4 Joint Input-Output Efficiency 153

9.5 Computing Input Efficiency 154

9.5.1 *CRS* Technology 154

9.5.2 *VRS* Technology 156

9.5.3 *HR* Technology 156

9.6 Computing Output Efficiency 157

9.7 Computing Cost Efficiency 158

9.8 Computing Joint Input-Output Efficiency 158

9.9 Exercises 159

9.10 Bibliographical Notes 161

9.11 Solutions to Exercises 162

10 The Two-Dimensional Projection 167

10.1 Definition 167

10.2 Characterizations 168

10.3 Computing Efficiency 171

10.4 Scale Characterizations 172

10.5 Example 172

10.6 Extensions 175

10.7 Pivoting Algorithm 176

10.7.1 Vertices and the Simplex Tableau 177

10.7.2 Pivot Operation 178

10.7.3 Phase I 180

10.7.4 Phase II 181

10.8 Exercises 183

10.9 Bibliographical Notes 185

10.10 Solutions to Exercises 186

11 Multi-Stage Efficiency Analysis 191

11.1 A Representative Multi-Stage System 192

11.2 Description of Multi-Stage Technology 193

11.2.1 Classical Models of Technology 193

11.2.2 Expanded Model of Technology 194

11.2.3 Expanded Subsystem Technology Sets 196

11.3 Pareto efficient Frontiers 197

11.4 Aggregate Efficiency 199

11.4.1 Measures of Aggregate Input Efficiency 199

11.4.2 Derived Measure of Aggregate Efficiency 200

11.4.3	Computational Results	201
11.5	A Consistent Pricing Principle	203
11.6	Extensions	205
11.7	Bibliographical Notes	205
12	Efficiency Analysis of Warehouse and Distribution Operations	207
12.1	Business Environment	207
12.2	Description of Technology	208
12.2.1	Input Categories	208
12.2.2	Output Categories	209
12.2.3	Caveats	211
12.3	Measuring Operating Efficiency	211
12.4	Empirical Results	214
12.5	Current Assessment	215
12.6	Data and Results	216
12.7	Exercises	220
12.8	Bibliographical Notes	220
<hr/>		
Part III Productivity and Performance Measurement		
<hr/>		
13	Index Numbers	223
13.1	Motivating Example	223
13.2	Price Indexes	227
13.2.1	Konus Price Index	227
13.2.2	Laspeyres and Paasche Price Indexes	228
13.3	Fisher and Tornqvist Price Indexes	230
13.3.1	Fisher Ideal Price Index	230
13.3.2	Tornqvist Price Index	231
13.4	Implicit Quantity Indexes	233
13.5	Quantity Indexes	233
13.6	Implicit Price Indexes	234
13.7	Exercises	235
13.8	Bibliographical Notes	237
13.9	Solutions to Exercises	238
14	Productivity Measurement	241
14.1	Growth Rates	241
14.2	Growth Accounting Approach	243
14.3	Multi-Output Productivity Measurement	245
14.4	Nonparametric Approach	246
14.4.1	Input Productivity Change	246
14.4.2	Output Productivity Change	248
14.5	Exercises	250

14.6 Bibliographical Notes 252

14.7 Solutions to Exercises 253

15 Performance Measurement 257

15.1 A Manufacturing Example 257

15.2 Performance Indexes 259

15.3 Productivity Assessment 261

15.4 Performance Ratios 262

15.4.1 Profitability Ratio 263

15.4.2 Productivity Ratio 264

15.4.3 Price Recovery Ratio 264

15.5 Distribution of Net Gain 265

15.5.1 Net Gain 266

15.5.2 Net Gain Due to Productivity 267

15.5.3 Net Gain Due to Price Recovery 267

15.6 Exercises 268

15.7 Bibliographical Notes 268

15.8 Solutions to Exercises 269

16 Economic Analysis 271

16.1 Market Structure and Equilibrium 271

16.2 Competitive Market Structure 273

16.2.1 Consumers 273

16.2.2 Producers 274

16.2.3 Equilibrium 274

16.2.4 Comparative Statics 276

16.3 Monopolistic Competitive Market Structure 277

16.4 Social Planner’s Perspective 278

16.5 Oligopoly Market Structure 279

16.5.1 Profit Maximization Formulation 279

16.5.2 Equilibrium 280

16.5.3 Algorithm to Compute the Equilibrium 282

16.5.4 Comparison to Competitive and Monopolistic
Competitive Market Structures 283

16.6 Productivity Analysis 284

16.6.1 Analysis of a Productivity Laggard 284

16.6.2 Analysis of a Productivity Leader 284

16.7 Exercises 284

16.8 Bibliographical Notes 287

16.9 Solutions to Exercises 288

Part IV Engineering Models of Technology

17	Index-Based Dynamic Production Functions	295
17.1	A Motivating Example	295
17.2	Input-Output Domain	297
17.2.1	Event-Based Flows	298
17.2.2	Rate-Based Flows	298
17.3	Instantaneous Processes	298
17.4	Index-Based Processes	299
17.4.1	Definition	299
17.4.2	Fixed Proportions, Instantaneous Model	300
17.4.3	Fixed Proportions, Constant Lead Time Models	300
17.5	Exercises	304
17.6	Bibliographical Notes	305
17.7	Solutions to Exercises	306
18	Distribution-Based Dynamic Production Functions	309
18.1	Description	309
18.1.1	Overview	309
18.1.2	Definition	310
18.1.3	Lead Time Density	311
18.1.4	Technical Remarks	312
18.2	Constant Lead Time Processes	313
18.2.1	Description	313
18.2.2	Integer Lead Times	314
18.2.3	Noninteger Lead Times	315
18.2.4	Non-Integer Lead Times with Unequal Length Periods	318
18.3	Time-Dependent Lead Time Processes	320
18.3.1	Description	320
18.3.2	First-In, First-Out Example	321
18.3.3	Leapfrog Example	322
18.4	Continuous Lead Time Processes	324
18.4.1	Description	324
18.4.2	Examples	326
18.5	Exercises	329
18.6	Solutions to Exercises	331
19	Dynamic Production Function Approximations	337
19.1	Load-Dependent Processes	337
19.1.1	Formulation	338
19.1.2	Example	339
19.1.3	Linear Approximation	340
19.1.4	Load-Dependent, Linear Approximation	346

- 19.2 Two-Point Boundary Approximation 348
 - 19.2.1 Relative Area Ratio 349
 - 19.2.2 Linear Approximation 350
 - 19.2.3 Example 351
 - 19.2.4 Extensions 353
- 19.3 Application to Project-Oriented Production Systems 356
 - 19.3.1 Description 356
 - 19.3.2 Detailed Activities 357
 - 19.3.3 Aggregate Activities 358
 - 19.3.4 Aggregate Dynamic Production Function 360
- 19.4 Aggregation of Dynamic Production Functions 361
 - 19.4.1 Serial Aggregation 361
 - 19.4.2 Parallel Aggregation 362
- 19.5 Estimation via Dynamic Activity Analysis 362
 - 19.5.1 Basic Model 362
 - 19.5.2 Extensions 363
- 19.6 Exercises 364
- 19.7 Bibliographical Notes 365
- 19.8 Solutions to Exercises 366

- 20 A Stochastic Input-Output Model 373**
 - 20.1 Input-Output Model with Single Inputs 373
 - 20.2 Input-Output Model with Batch Input 375
 - 20.2.1 Simultaneous Batch Case 376
 - 20.2.2 Independent Batch Case 377
 - 20.3 Confidence intervals 378
 - 20.3.1 Without Batch Input 378
 - 20.3.2 With Batch Input 379
 - 20.3.3 Linear Approximation 382
 - 20.4 Exercises 383
 - 20.5 Bibliographical Notes 385
 - 20.6 Solutions to Exercises 386

- 21 Multi-Stage, Dynamic Models of Technology 391**
 - 21.1 Basic Model 392
 - 21.1.1 Primitives 392
 - 21.1.2 Material Balance and Service Capacity Constraints ... 393
 - 21.2 Index-Based Models 394
 - 21.2.1 Instantaneous Processes 394
 - 21.2.2 Constant Lead Time Processes 395
 - 21.2.3 Multi-Event, Constant Lead Time Processes 396
 - 21.2.4 Continuous Lead Time Based Processes 397
 - 21.2.5 Initial Conditions 398
 - 21.3 Computational Models 401

21.4 A Manufacturing Example 404
 21.4.1 Production Process Description 404
 21.4.2 Formulation 404
 21.4.3 Extensions 407
 21.5 Assembly with Rework Example 408
 21.5.1 Production Process Description 408
 21.5.2 Formulation 409
 21.5.3 Extensions 411
 21.6 Extensions to the Basic Model 411
 21.6.1 Material Balance Constraints 411
 21.6.2 Transfers of Product or Materials 412
 21.6.3 Activity Constraints 412
 21.6.4 Service Output 413
 21.6.5 Alternate Production Processes 413
 21.6.6 Load-Dependent, Multi-Product, Single-Stage Model . . 414
 21.7 Efficiency and Productivity Measurement 417
 21.7.1 Input and Output Efficiency 417
 21.7.2 Cost and Allocative Efficiency 417
 21.7.3 Productivity Assessment 418
 21.7.4 Computation 418
 21.8 Bibliographical Notes 418

22 Optimizing Labor Resources Within a Warehouse 421
 22.1 Introduction 421
 22.2 System Description 422
 22.2.1 Business Environment 422
 22.2.2 Material Flow 423
 22.2.3 Workforce Schedule 423
 22.2.4 Sources of Inefficiency 423
 22.3 An Optimization Model 425
 22.3.1 Parameters 425
 22.3.2 Decision Variables 426
 22.3.3 Constraints 427
 22.3.4 Objective Function 428
 22.4 Implementation 429
 22.4.1 Computational Issues 429
 22.4.2 Using the Prototype Model: A Case Study 430
 22.4.3 Benefits and Other Applications 430
 22.5 Bibliographical Notes 431

Part V Mathematical Appendix

A	Notation and Mathematical Preliminaries	435
A.1	Logical Statements	435
A.2	Sets	435
A.3	Vectors	438
A.4	Correspondences	439
A.5	Functions	440
A.6	Matrices	442
A.7	Differentiability	444
B	Real Analysis	449
B.1	Linear Spaces	449
B.1.1	Definition	449
B.1.2	Examples	450
B.2	Linear Independence and Dimension	451
B.3	Normed Linear Spaces	451
B.3.1	Definition	451
B.3.2	Examples	452
B.4	Metric Spaces	453
B.4.1	Definition	453
B.4.2	Open and Closed Sets	454
B.4.3	Closure and Boundary	454
B.4.4	Convergence and Limits	456
B.4.5	Completeness	456
B.4.6	Compactness	457
B.4.7	Continuity	458
B.4.8	Connectedness	460
B.5	Bibliographical Notes	460
C	Convex Sets	461
C.1	Definition and Examples	461
C.2	Convexification	462
C.3	Separation of a Convex Set and a Point	463
C.3.1	Strict Separation	463
C.3.2	Supporting Hyperplanes	464
C.3.3	Polar Cones	464
C.4	Polyhedra	465
C.4.1	Definition and Examples	465
C.4.2	Extreme Points and Directions	466
C.4.3	Characterization of Extreme Points and Directions	467
C.4.4	Representation Theorem for Polyhedra	470
C.5	Application to Linear Programming	471
C.6	Bibliographical Notes	472

D Concave, Convex Functions and Generalizations 473

D.1 Definitions 473

D.2 Quasiconcavity and Quasiconvexity 474

D.3 Differential Characterizations 476

E Optimality Conditions 479

E.1 Unconstrained Problems 479

E.2 Problems with Inequality Constraints 480

E.3 Lagrangian Duality 482

E.4 Application of Duality to Economic Lot Sizes 486

E.5 Application of Duality to Linear Programming 487

E.6 Bibliographical Notes 489

F Envelope Theorem 491

F.1 Statement and Proof 491

F.2 Application to Sensitivity Analysis of Cost 493

F.3 A Monopoly Pricing Example 493

F.4 Bibliographical Notes 494

G Correspondence Theory 495

G.1 Core Concepts 495

G.2 Characterization by Sequences 498

G.3 Bibliographical Notes 499

H Theorem of the Maximum 501

H.1 Application to the Indirect Production Function 502

H.2 Application to the Cost Function 503

H.3 Bibliographical Notes 505

I Probability Basics 507

I.1 Binomial Random Variables 507

I.2 Poisson Random Variables 507

I.3 Poisson Processes 508

I.4 Moment Generating Functions 509

I.5 Conditional Expectation and Variance 509

I.6 Bibliographical Notes 510

References 511

Index 517