
Contents

1	Introduction	1
----------	---------------------	----------

Part I The Method of Intrinsic Scaling

2	Weak Solutions and <i>a Priori</i> Estimates	11
2.1	Definition of Weak Solution	11
2.2	Local Energy Estimates: The Building Blocks of the Theory	13
2.3	Local Logarithmic Estimates	15
2.4	Some Technical Tools	17
3	The Geometric Setting and an Alternative	21
3.1	A Geometry for the Equation	22
3.2	The First Alternative	25
3.3	The Role of the Logarithmic Estimates: Expansion in Time	28
3.4	Reduction of the Oscillation	31
4	Towards the Hölder Continuity	35
4.1	Expanding in Time	35
4.2	Reducing the Oscillation	39
4.3	Defining the Geometry	41
4.4	The Recursive Argument	44
4.5	Generalizations	47

Part II Some Applications

5	Immiscible Fluids and Chemotaxis	51
5.1	The Flow of Two Immiscible Fluids through a Porous Medium	51
5.2	Rescaled Cylinders	53
5.3	Focusing on One Degeneracy	55

X Contents

5.4	Behaviour Near the other Degeneracy	67
5.5	A Problem in Chemotaxis	81
6	Flows in Porous Media: The Variable Exponent Case	87
6.1	The Porous Medium Equation in its Own Geometry	87
6.2	Reducing the Oscillation	89
6.3	Analysis of the Alternative	95
7	Phase Transitions: The Doubly Singular Stefan Problem	107
7.1	Regularization of the Maximal Monotone Graph.....	108
7.2	A Third Power in the Energy Estimates	110
7.3	The Intrinsic Geometry	112
7.4	Analyzing the Singularity in Time	117
7.5	The Effect of the Singularity in the Principal Part	126
7.5.1	An Equation in Dimensionless Form	130
7.5.2	Expansion in Space	139
	References	145
	Index	149