

---

# Contents

<b>1</b>	<b>Appetizer: Integer Arithmetics</b>	1
1.1	Addition	2
1.2	Multiplication: The School Method	3
1.3	Result Checking	6
1.4	A Recursive Version of the School Method	7
1.5	Karatsuba Multiplication	9
1.6	Algorithm Engineering	11
1.7	The Programs	13
1.8	Proofs of Lemma 1.5 and Theorem 1.7	16
1.9	Implementation Notes	17
1.10	Historical Notes and Further Findings	18
<b>2</b>	<b>Introduction</b>	19
2.1	Asymptotic Notation	20
2.2	The Machine Model	23
2.3	Pseudocode	26
2.4	Designing Correct Algorithms and Programs	31
2.5	An Example – Binary Search	34
2.6	Basic Algorithm Analysis	36
2.7	Average-Case Analysis	41
2.8	Randomized Algorithms	45
2.9	Graphs	49
2.10	<b>P</b> and <b>NP</b>	53
2.11	Implementation Notes	56
2.12	Historical Notes and Further Findings	57
<b>3</b>	<b>Representing Sequences by Arrays and Linked Lists</b>	59
3.1	Linked Lists	60
3.2	Unbounded Arrays	66
3.3	*Amortized Analysis	71
3.4	Stacks and Queues	74

3.5	Lists Versus Arrays . . . . .	77
3.6	Implementation Notes . . . . .	78
3.7	Historical Notes and Further Findings . . . . .	79
<b>4</b>	<b>Hash Tables and Associative Arrays . . . . .</b>	<b>81</b>
4.1	Hashing with Chaining . . . . .	83
4.2	Universal Hashing . . . . .	85
4.3	Hashing with Linear Probing . . . . .	90
4.4	Chaining Versus Linear Probing . . . . .	92
4.5	*Perfect Hashing . . . . .	92
4.6	Implementation Notes . . . . .	95
4.7	Historical Notes and Further Findings . . . . .	97
<b>5</b>	<b>Sorting and Selection . . . . .</b>	<b>99</b>
5.1	Simple Sorters . . . . .	101
5.2	Mergesort – an $O(n \log n)$ Sorting Algorithm . . . . .	103
5.3	A Lower Bound . . . . .	106
5.4	Quicksort . . . . .	108
5.5	Selection . . . . .	114
5.6	Breaking the Lower Bound . . . . .	116
5.7	*External Sorting . . . . .	118
5.8	Implementation Notes . . . . .	122
5.9	Historical Notes and Further Findings . . . . .	124
<b>6</b>	<b>Priority Queues . . . . .</b>	<b>127</b>
6.1	Binary Heaps . . . . .	129
6.2	Addressable Priority Queues . . . . .	133
6.3	*External Memory . . . . .	139
6.4	Implementation Notes . . . . .	141
6.5	Historical Notes and Further Findings . . . . .	142
<b>7</b>	<b>Sorted Sequences . . . . .</b>	<b>145</b>
7.1	Binary Search Trees . . . . .	147
7.2	$(a, b)$ -Trees and Red–Black Trees . . . . .	149
7.3	More Operations . . . . .	156
7.4	Amortized Analysis of Update Operations . . . . .	158
7.5	Augmented Search Trees . . . . .	160
7.6	Implementation Notes . . . . .	162
7.7	Historical Notes and Further Findings . . . . .	164
<b>8</b>	<b>Graph Representation . . . . .</b>	<b>167</b>
8.1	Unordered Edge Sequences . . . . .	168
8.2	Adjacency Arrays – Static Graphs . . . . .	168
8.3	Adjacency Lists – Dynamic Graphs . . . . .	170
8.4	The Adjacency Matrix Representation . . . . .	171
8.5	Implicit Representations . . . . .	172

8.6	Implementation Notes . . . . .	172
8.7	Historical Notes and Further Findings . . . . .	174
<b>9</b>	<b>Graph Traversal . . . . .</b>	<b>175</b>
9.1	Breadth-First Search . . . . .	176
9.2	Depth-First Search . . . . .	178
9.3	Implementation Notes . . . . .	188
9.4	Historical Notes and Further Findings . . . . .	189
<b>10</b>	<b>Shortest Paths . . . . .</b>	<b>191</b>
10.1	From Basic Concepts to a Generic Algorithm . . . . .	192
10.2	Directed Acyclic Graphs . . . . .	195
10.3	Nonnegative Edge Costs (Dijkstra’s Algorithm) . . . . .	196
10.4	*Average-Case Analysis of Dijkstra’s Algorithm . . . . .	199
10.5	Monotone Integer Priority Queues . . . . .	201
10.6	Arbitrary Edge Costs (Bellman–Ford Algorithm) . . . . .	206
10.7	All-Pairs Shortest Paths and Node Potentials . . . . .	207
10.8	Shortest-Path Queries . . . . .	209
10.9	Implementation Notes . . . . .	213
10.10	Historical Notes and Further Findings . . . . .	214
<b>11</b>	<b>Minimum Spanning Trees . . . . .</b>	<b>217</b>
11.1	Cut and Cycle Properties . . . . .	218
11.2	The Jarník–Prim Algorithm . . . . .	219
11.3	Kruskal’s Algorithm . . . . .	221
11.4	The Union–Find Data Structure . . . . .	222
11.5	*External Memory . . . . .	225
11.6	Applications . . . . .	228
11.7	Implementation Notes . . . . .	231
11.8	Historical Notes and Further Findings . . . . .	231
<b>12</b>	<b>Generic Approaches to Optimization . . . . .</b>	<b>233</b>
12.1	Linear Programming – a Black-Box Solver . . . . .	234
12.2	Greedy Algorithms – Never Look Back . . . . .	239
12.3	Dynamic Programming – Building It Piece by Piece . . . . .	243
12.4	Systematic Search – When in Doubt, Use Brute Force . . . . .	246
12.5	Local Search – Think Globally, Act Locally . . . . .	249
12.6	Evolutionary Algorithms . . . . .	259
12.7	Implementation Notes . . . . .	261
12.8	Historical Notes and Further Findings . . . . .	262
<b>A</b>	<b>Appendix . . . . .</b>	<b>263</b>
A.1	Mathematical Symbols . . . . .	263
A.2	Mathematical Concepts . . . . .	264
A.3	Basic Probability Theory . . . . .	266
A.4	Useful Formulae . . . . .	269

<b>References</b> .....	273
<b>Index</b> .....	285