
Contents

Preface	V
Lectures on Gromov–Witten Invariants of Orbifolds	
<i>D. Abramovich</i>	1
1 Introduction	1
1.1 What This Is	1
1.2 Introspection	1
1.3 Where Does All This Come From?	2
1.4 Acknowledgements	2
2 Gromov–Witten Theory	2
2.1 Kontsevich’s Formula	2
2.2 Set-Up for a Streamlined Proof	3
2.3 The Space of Stable Maps	7
2.4 Natural Maps	8
2.5 Boundary of Moduli	9
2.6 Gromov–Witten Classes	10
2.7 The WDVV Equations	11
2.8 Proof of WDVV	12
2.9 About the General Case	15
3 Orbifolds/Stacks	16
3.1 Geometric Orbifolds	16
3.2 Moduli Stacks	17
3.3 Where Do Stacks Come Up?	19
3.4 Attributes of Orbifolds	19
3.5 Étale Gerbes	20
4 Twisted Stable Maps	21
4.1 Stable Maps to a Stack	21
4.2 Twisted Curves	22
4.3 Twisted Stable Maps	23
4.4 Transparency 25: The Stack of Twisted Stable Maps	24
4.5 Twisted Curves and Roots	25
4.6 Valuative Criterion for Properness	27

VIII Contents

5	Gromov–Witten Classes	29
5.1	Contractions	29
5.2	Gluing and Rigidified Inertia	29
5.3	Evaluation Maps	31
5.4	The Boundary of Moduli	32
5.5	Orbifold Gromov–Witten Classes	32
5.6	Fundamental Classes	34
6	WDVV, Grading and Computations	35
6.1	The Formula	35
6.2	Quantum Cohomology and Its Grading	36
6.3	Grading the Rings	38
6.4	Examples	38
6.5	Other Work	41
6.6	Mirror Symmetry and the Crepant Resolution Conjecture	42
A	The Legend of String Cohomology: Two Letters of Maxim Kontsevich to Lev Borisov	43
A.1	The Legend of String Cohomology	43
A.2	The Archaeological Letters	44
	References	46

Lectures on the Topological Vertex

<i>M. Mariño</i>	49
1	Introduction and Overview	49
2	Chern–Simons Theory	51
2.1	Basic Ingredients	51
2.2	Perturbative Approach	55
2.3	Non-Perturbative Solution	61
2.4	Framing Dependence	68
2.5	The $1/N$ Expansion in Chern–Simons Theory	70
3	Topological Strings	73
3.1	Topological Strings and Gromov–Witten Invariants	74
3.2	Integrality Properties and Gopakumar–Vafa Invariants	76
3.3	Open Topological Strings	77
4	Toric Geometry and Calabi–Yau Threefolds	79
4.1	Non-Compact Calabi–Yau Geometries: An Introduction	79
4.2	Constructing Toric Calabi–Yau Manifolds	81
4.3	Examples of Closed String Amplitudes	87
5	The Topological Vertex	89
5.1	The Gopakumar–Vafa Duality	89
5.2	Framing of Topological Open String Amplitudes	89
5.3	Definition of the Topological Vertex	91
5.4	Gluing Rules	93
5.5	Explicit Expression for the Topological Vertex	95
5.6	Applications	96
A	Symmetric Polynomials	99
	References	100

Floer Cohomology with Gerbes

M. Thaddeus 105

1 Floer Cohomology 106

 1.1 Newton’s Second Law 106

 1.2 The Hamiltonian Formalism 107

 1.3 The Arnold Conjecture 108

 1.4 Floer’s Proof 108

 1.5 Morse Theory 109

 1.6 Bott–Morse Theory 110

 1.7 Morse Theory on the Loop Space 110

 1.8 Re-Interpretation #1: Sections of the Symplectic Mapping Torus 112

 1.9 Re-Interpretation #2: Two Lagrangian Submanifolds 113

 1.10 Product Structures 114

 1.11 The Finite-Order Case 115

 1.12 Givental’s Philosophy 115

2 Gerbes 117

 2.1 Definition of Stacks 117

 2.2 Examples of Stacks 118

 2.3 Morphisms and 2-Morphisms 118

 2.4 Definition of Gerbes 120

 2.5 The Gerbe of Liftings 121

 2.6 The Lien of a Gerbe 122

 2.7 Classification of Gerbes 123

 2.8 Allowing the Base Space to Be a Stack 123

 2.9 Definition of Orbifolds 124

 2.10 Twisted Vector Bundles 124

 2.11 Strominger–Yau–Zaslow 125

3 Orbifold Cohomology and Its Relatives 126

 3.1 Cohomology of Sheaves on Stacks 126

 3.2 The Inertia Stack 127

 3.3 Orbifold Cohomology 128

 3.4 Twisted Orbifold Cohomology 129

 3.5 The Case of Discrete Torsion 129

 3.6 The Fantechi–Göttsche Ring 130

 3.7 Twisting the Fantechi–Göttsche Ring with Discrete Torsion 131

 3.8 Twisting It with an Arbitrary Flat Unitary Gerbe 131

 3.9 The Loop Space of an Orbifold 132

 3.10 Addition of the Gerbe 134

 3.11 The Non-Orbifold Case 135

 3.12 The Equivariant Case 135

 3.13 A Concluding Puzzle 136

4 Notes on the Literature 137

 4.1 Notes to Lecture 1 137

 4.2 Notes to Lecture 2 139

 4.3 Notes to Lecture 3 140

The Moduli Space of Curves and Gromov–Witten Theory

<i>R. Vakil</i>	143
1 Introduction	143
2 The Moduli Space of Curves	145
3 Tautological Cohomology Classes on Moduli Spaces of Curves, and Their Structure	154
4 A Blunt Tool: Theorem \star and Consequences	173
5 Stable Relative Maps to \mathbb{P}^1 and Relative Virtual Localization	177
6 Applications of Relative Virtual Localization	186
7 Towards Faber’s Intersection Number Conjecture 3.23 via Relative Virtual Localization	190
8 Conclusion	194
References	194
List of Participants	199