

Contents

Part I. Introduction

Prologue.....	1
1. Mathematical Principles of Modern Natural Philosophy .	11
1.1 Basic Principles	12
1.2 The Infinitesimal Strategy and Differential Equations.....	14
1.3 The Optimality Principle	14
1.4 The Basic Notion of Action in Physics and the Idea of Quantization	15
1.5 The Method of the Green's Function	17
1.6 Harmonic Analysis and the Fourier Method	21
1.7 The Method of Averaging and the Theory of Distributions .	26
1.8 The Symbolic Method	28
1.9 Gauge Theory – Local Symmetry and the Description of Interactions by Gauge Fields	34
1.10 The Challenge of Dark Matter	46
2. The Basic Strategy of Extracting Finite Information from Infinites – Ariadne's Thread in Renormalization Theory	47
2.1 Renormalization Theory in a Nutshell	48
2.1.1 Effective Frequency and Running Coupling Constant of an Anharmonic Oscillator	48
2.1.2 The Zeta Function and Riemann's Idea of Analytic Continuation	54
2.1.3 Meromorphic Functions and Mittag-Leffler's Idea of Subtractions	56
2.1.4 The Square of the Dirac Delta Function	58
2.2 Regularization of Divergent Integrals in Baby Renormalization Theory	60
2.2.1 Momentum Cut-off and the Method of Power-Counting	60
2.2.2 The Choice of the Normalization Momentum	63
2.2.3 The Method of Differentiating Parameter Integrals .	63
2.2.4 The Method of Taylor Subtraction	64

2.2.5	Overlapping Divergences	65
2.2.6	The Role of Counterterms	67
2.2.7	Euler's Gamma Function	67
2.2.8	Integration Tricks	69
2.2.9	Dimensional Regularization via Analytic Continuation	73
2.2.10	Pauli–Villars Regularization	76
2.2.11	Analytic Regularization	77
2.2.12	Application to Algebraic Feynman Integrals in Minkowski Space	80
2.2.13	Distribution-Valued Meromorphic Functions	81
2.2.14	Application to Newton's Equation of Motion	87
2.2.15	Hints for Further Reading.	92
2.3	Further Regularization Methods in Mathematics	93
2.3.1	Euler's Philosophy	93
2.3.2	Adiabatic Regularization of Divergent Series	94
2.3.3	Adiabatic Regularization of Oscillating Integrals	95
2.3.4	Regularization by Averaging	96
2.3.5	Borel Regularization	98
2.3.6	Hadamard's Finite Part of Divergent Integrals	100
2.3.7	Infinite-Dimensional Gaussian Integrals and the Zeta Function Regularization	101
2.4	Trouble in Mathematics	102
2.4.1	Interchanging Limits	102
2.4.2	The Ambiguity of Regularization Methods	104
2.4.3	Pseudo-Convergence	104
2.4.4	Ill-Posed Problems	105
2.5	Mathemagics	109
3.	The Power of Combinatorics	115
3.1	Algebras	115
3.2	The Algebra of Multilinear Functionals	117
3.3	Fusion, Splitting, and Hopf Algebras	122
3.3.1	The Bialgebra of Linear Differential Operators	123
3.3.2	The Definition of Hopf Algebras	128
3.4	Power Series Expansion and Hopf Algebras	131
3.4.1	The Importance of Cancellations	131
3.4.2	The Kepler Equation and the Lagrange Inversion Formula	132
3.4.3	The Composition Formula for Power Series	134
3.4.4	The Faà di Bruno Hopf Algebra for the Formal Diffeomorphism Group of the Complex Plane	136
3.4.5	The Generalized Zimmermann Forest Formula	138
3.4.6	The Logarithmic Function and Schur Polynomials	140
3.4.7	Correlation Functions in Quantum Field Theory	141

3.4.8	Random Variables, Moments, and Cumulants	143
3.5	Symmetry and Hopf Algebras	146
3.5.1	The Strategy of Coordinatization in Mathematics and Physics	146
3.5.2	The Coordinate Hopf Algebra of a Finite Group . . .	148
3.5.3	The Coordinate Hopf Algebra of an Operator Group	150
3.5.4	The Tannaka–Krein Duality for Compact Lie Groups	152
3.6	Regularization and Rota–Baxter Algebras	154
3.6.1	Regularization of the Laurent Series	157
3.6.2	Projection Operators	158
3.6.3	The q -Integral	158
3.6.4	The Volterra–Spitzer Exponential Formula	160
3.6.5	The Importance of the Exponential Function in Mathematics and Physics	161
3.7	Partially Ordered Sets and Combinatorics	162
3.7.1	Incidence Algebras and the Zeta Function	162
3.7.2	The Möbius Function as an Inverse Function	163
3.7.3	The Inclusion–Exclusion Principle in Combinatorics	164
3.7.4	Applications to Number Theory	166
3.8	Hints for Further Reading	167
4.	The Strategy of Equivalence Classes in Mathematics	175
4.1	Equivalence Classes in Algebra	178
4.1.1	The Gaussian Quotient Ring and the Quadratic Reciprocity Law in Number Theory	178
4.1.2	Application of the Fermat–Euler Theorem in Coding Theory	182
4.1.3	Quotient Rings, Quotient Groups, and Quotient Fields	184
4.1.4	Linear Quotient Spaces	188
4.1.5	Ideals and Quotient Algebras	190
4.2	Superfunctions and the Heaviside Calculus in Electrical Engineering	191
4.3	Equivalence Classes in Geometry	194
4.3.1	The Basic Idea of Geometry Epitomized by Klein’s Erlangen Program	194
4.3.2	Symmetry Spaces, Orbit Spaces, and Homogeneous Spaces	194
4.3.3	The Space of Quantum States	199
4.3.4	Real Projective Spaces	200
4.3.5	Complex Projective Spaces	203
4.3.6	The Shape of the Universe	204
4.4	Equivalence Classes in Topology	205
4.4.1	Topological Quotient Spaces	205
4.4.2	Physical Fields, Observers, Bundles, and Cocycles . .	208
4.4.3	Generalized Physical Fields and Sheaves	216

4.4.4	Deformations, Mapping Classes, and Topological Charges	219
4.4.5	Poincaré’s Fundamental Group	223
4.4.6	Loop Spaces and Higher Homotopy Groups	225
4.4.7	Homology, Cohomology, and Electrodynamics	227
4.4.8	Bott’s Periodicity Theorem	227
4.4.9	K -Theory	228
4.4.10	Application to Fredholm Operators	233
4.4.11	Hints for Further Reading	235
4.5	The Strategy of Partial Ordering	237
4.5.1	Feynman Diagrams	238
4.5.2	The Abstract Entropy Principle in Thermodynamics	239
4.5.3	Convergence of Generalized Sequences	240
4.5.4	Inductive and Projective Topologies	241
4.5.5	Inductive and Projective Limits	243
4.5.6	Classes, Sets, and Non-Sets	245
4.5.7	The Fixed-Point Theorem of Bourbaki–Kneser	247
4.5.8	Zorn’s Lemma	248
4.6	Leibniz’s Infinitesimals and Non-Standard Analysis	248
4.6.1	Filters and Ultrafilters	250
4.6.2	The Full-Rigged Real Line	251

Part II. Basic Ideas in Classical Mechanics

5.	Geometrical Optics	263
5.1	Ariadne’s Thread in Geometrical Optics	264
5.2	Fermat’s Principle of Least Time	268
5.3	Huygens’ Principle on Wave Fronts	270
5.4	Carathéodory’s Royal Road to Geometrical Optics	271
5.5	The Duality between Light Rays and Wave Fronts	274
5.5.1	From Wave Fronts to Light Rays	275
5.5.2	From Light Rays to Wave Fronts	276
5.6	The Jacobi Approach to Focal Points	276
5.7	Lie’s Contact Geometry	279
5.7.1	Basic Ideas	279
5.7.2	Contact Manifolds and Contact Transformations	283
5.7.3	Applications to Geometrical Optics	284
5.7.4	Equilibrium Thermodynamics and Legendre Submanifolds	285
5.8	Light Rays and Non-Euclidean Geometry	289
5.8.1	Linear Symplectic Spaces	291
5.8.2	The Kähler Form of a Complex Hilbert Space	295
5.8.3	The Refraction Index and Geodesics	297
5.8.4	The Trick of Gauge Fixing	299

5.8.5	Geodesic Flow	299
5.8.6	Hamilton’s Duality Trick and Cogeodesic Flow	300
5.8.7	The Principle of Minimal Geodesic Energy	301
5.9	Spherical Geometry	302
5.9.1	The Global Gauss–Bonnet Theorem	303
5.9.2	De Rham Cohomology and the Chern Class of the Sphere	305
5.9.3	The Beltrami Model	308
5.10	The Poincaré Model of Hyperbolic Geometry	314
5.10.1	Kähler Geometry and the Gaussian Curvature	318
5.10.2	Kähler–Einstein Geometry	323
5.10.3	Symplectic Geometry	323
5.10.4	Riemannian Geometry	324
5.11	Ariadne’s Thread in Gauge Theory	333
5.11.1	Parallel Transport of Physical Information – the Key to Modern Physics	334
5.11.2	The Phase Equation and Fiber Bundles	337
5.11.3	Gauge Transformations and Gauge-Invariant Differential Forms	338
5.11.4	Perspectives	341
5.12	Classification of Two-Dimensional Compact Manifolds	343
5.13	The Poincaré Conjecture and the Ricci Flow	346
5.14	A Glance at Modern Optimization Theory	348
5.15	Hints for Further Reading	348
6.	The Principle of Critical Action and the Harmonic Oscillator – Ariadne’s Thread in Classical Mechanics	359
6.1	Prototypes of Extremal Problems	360
6.2	The Motion of a Particle	364
6.3	Newtonian Mechanics	366
6.4	A Glance at the History of the Calculus of Variations	370
6.5	Lagrangian Mechanics	372
6.5.1	The Harmonic Oscillator	373
6.5.2	The Euler–Lagrange Equation	375
6.5.3	Jacobi’s Accessory Eigenvalue Problem	376
6.5.4	The Morse Index	377
6.5.5	The Anharmonic Oscillator	378
6.5.6	The Ginzburg–Landau Potential and the Higgs Potential	380
6.5.7	Damped Oscillations, Stability, and Energy Dissipation	382
6.5.8	Resonance and Small Divisors	382
6.6	Symmetry and Conservation Laws	383
6.6.1	The Symmetries of the Harmonic Oscillator	384
6.6.2	The Noether Theorem	384

6.7	The Pendulum and Dynamical Systems	390
6.7.1	The Equation of Motion	390
6.7.2	Elliptic Integrals and Elliptic Functions	391
6.7.3	The Phase Space of the Pendulum and Bundles	396
6.8	Hamiltonian Mechanics	402
6.8.1	The Canonical Equation	404
6.8.2	The Hamiltonian Flow	404
6.8.3	The Hamilton–Jacobi Partial Differential Equation	405
6.9	Poissonian Mechanics	406
6.9.1	Poisson Brackets and the Equation of Motion	407
6.9.2	Conservation Laws	407
6.10	Symplectic Geometry	407
6.10.1	The Canonical Equations	408
6.10.2	Symplectic Transformations	409
6.11	The Spherical Pendulum	411
6.11.1	The Gaussian Principle of Critical Constraint	411
6.11.2	The Lagrangian Approach	412
6.11.3	The Hamiltonian Approach	414
6.11.4	Geodesics of Shortest Length	415
6.12	The Lie Group $SU(E^3)$ of Rotations	416
6.12.1	Conservation of Angular Momentum	416
6.12.2	Lie’s Momentum Map	419
6.13	Carathéodory’s Royal Road to the Calculus of Variations	419
6.13.1	The Fundamental Equation	419
6.13.2	Lagrangian Submanifolds in Symplectic Geometry	421
6.13.3	The Initial-Value Problem for the Hamilton–Jacobi Equation	423
6.13.4	Solution of Carathéodory’s Fundamental Equation	423
6.14	Hints for Further Reading	424

Part III. Basic Ideas in Quantum Mechanics

7.	Quantization of the Harmonic Oscillator – Ariadne’s Thread in Quantization	427
7.1	Complete Orthonormal Systems	430
7.2	Bosonic Creation and Annihilation Operators	432
7.3	Heisenberg’s Quantum Mechanics	440
7.3.1	Heisenberg’s Equation of Motion	443
7.3.2	Heisenberg’s Uncertainty Inequality for the Harmonic Oscillator	446
7.3.3	Quantization of Energy	447
7.3.4	The Transition Probabilities	449
7.3.5	The Wightman Functions	451
7.3.6	The Correlation Functions	456

7.4	Schrödinger's Quantum Mechanics	459
7.4.1	The Schrödinger Equation	459
7.4.2	States, Observables, and Measurements	462
7.4.3	The Free Motion of a Quantum Particle	464
7.4.4	The Harmonic Oscillator	467
7.4.5	The Passage to the Heisenberg Picture	473
7.4.6	Heisenberg's Uncertainty Principle	475
7.4.7	Unstable Quantum States and the Energy-Time Uncertainty Relation	476
7.4.8	Schrödinger's Coherent States	478
7.5	Feynman's Quantum Mechanics	479
7.5.1	Main Ideas	480
7.5.2	The Diffusion Kernel and the Euclidean Strategy in Quantum Physics	487
7.5.3	Probability Amplitudes and the Formal Propagator Theory	488
7.6	Von Neumann's Rigorous Approach	495
7.6.1	The Prototype of the Operator Calculus	496
7.6.2	The General Operator Calculus	499
7.6.3	Rigorous Propagator Theory	505
7.6.4	The Free Quantum Particle as a Paradigm of Functional Analysis	509
7.6.5	The Free Hamiltonian	524
7.6.6	The Rescaled Fourier Transform	532
7.6.7	The Quantized Harmonic Oscillator and the Maslov Index	534
7.6.8	Ideal Gases and von Neumann's Density Operator . .	540
7.7	The Feynman Path Integral	547
7.7.1	The Basic Strategy	547
7.7.2	The Basic Definition	549
7.7.3	Application to the Free Quantum Particle	550
7.7.4	Application to the Harmonic Oscillator	552
7.7.5	The Propagator Hypothesis	555
7.7.6	Motivation of Feynman's Path Integral	555
7.8	Finite-Dimensional Gaussian Integrals	559
7.8.1	Basic Formulas	560
7.8.2	Free Moments, the Wick Theorem, and Feynman Diagrams	564
7.8.3	Full Moments and Perturbation Theory	567
7.9	Rigorous Infinite-Dimensional Gaussian Integrals	570
7.9.1	The Infinite-Dimensional Dispersion Operator	571
7.9.2	Zeta Function Regularization and Infinite-Dimensional Determinants	572
7.9.3	Application to the Free Quantum Particle	574
7.9.4	Application to the Quantized Harmonic Oscillator . .	576

7.9.5	The Spectral Hypothesis	579
7.10	The Semi-Classical WKB Method	580
7.11	Brownian Motion	584
7.11.1	The Macroscopic Diffusion Law	584
7.11.2	Einstein's Key Formulas for the Brownian Motion ..	585
7.11.3	The Random Walk of Particles	585
7.11.4	The Rigorous Wiener Path Integral	586
7.11.5	The Feynman–Kac Formula	588
7.12	Weyl Quantization	590
7.12.1	The Formal Moyal Star Product	591
7.12.2	Deformation Quantization of the Harmonic Oscillator	592
7.12.3	Weyl Ordering	596
7.12.4	Operator Kernels	599
7.12.5	The Formal Weyl Calculus	602
7.12.6	The Rigorous Weyl Calculus	606
7.13	Two Magic Formulas	608
7.13.1	The Formal Feynman Path Integral for the Propaga- tor Kernel	611
7.13.2	The Relation between the Scattering Kernel and the Propagator Kernel	614
7.14	The Poincaré–Wirtinger Calculus	616
7.15	Bargmann's Holomorphic Quantization	617
7.16	The Stone–Von Neumann Uniqueness Theorem	621
7.16.1	The Prototype of the Weyl Relation	621
7.16.2	The Main Theorem	626
7.16.3	C^* -Algebras	627
7.16.4	Operator Ideals	629
7.16.5	Symplectic Geometry and the Weyl Quantization Functor	630
7.17	A Glance at the Algebraic Approach to Quantum Physics ..	633
7.17.1	States and Observables	633
7.17.2	Gleason's Extension Theorem – the Main Theorem of Quantum Logic	637
7.17.3	The Finite Standard Model in Statistical Physics as a Paradigm	638
7.17.4	Information, Entropy, and the Measure of Disorder .	640
7.17.5	Semiclassical Statistical Physics	645
7.17.6	The Classical Ideal Gas	648
7.17.7	Bose–Einstein Statistics	649
7.17.8	Fermi–Dirac Statistics	650
7.17.9	Thermodynamic Equilibrium and KMS-States	651
7.17.10	Quasi-Stationary Thermodynamic Processes and Irreversibility	652
7.17.11	The Photon Mill on Earth	654
7.18	Von Neumann Algebras	654

7.18.1	Von Neumann's Bicommutant Theorem	655
7.18.2	The Murray–von Neumann Classification of Factors	658
7.18.3	The Tomita–Takesaki Theory and KMS-States	659
7.19	Connes' Noncommutative Geometry	660
7.20	Jordan Algebras	662
7.21	The Supersymmetric Harmonic Oscillator	663
7.22	Hints for Further Reading	667
8.	Quantum Particles on the Real Line – Ariadne's Thread in Scattering Theory	699
8.1	Classical Dynamics Versus Quantum Dynamics	699
8.2	The Stationary Schrödinger Equation	703
8.3	One-Dimensional Quantum Motion in a Square-Well Potential	704
8.3.1	Free Motion	704
8.3.2	Scattering States and the S -Matrix	705
8.3.3	Bound States	710
8.3.4	Bound-State Energies and the Singularities of the S -Matrix	712
8.3.5	The Energetic Riemann Surface, Resonances, and the Breit–Wigner Formula	713
8.3.6	The Jost Functions	718
8.3.7	The Fourier–Stieltjes Transformation	719
8.3.8	Generalized Eigenfunctions of the Hamiltonian	720
8.3.9	Quantum Dynamics and the Scattering Operator	722
8.3.10	The Feynman Propagator	726
8.4	Tunnelling of Quantum Particles and Radioactive Decay	727
8.5	The Method of the Green's Function in a Nutshell	729
8.5.1	The Inhomogeneous Helmholtz Equation	730
8.5.2	The Retarded Green's Function, and the Existence and Uniqueness Theorem	731
8.5.3	The Advanced Green's Function	736
8.5.4	Perturbation of the Retarded and Advanced Green's Function	737
8.5.5	Feynman's Regularized Fourier Method	739
8.6	The Lippmann–Schwinger Integral Equation	743
8.6.1	The Born Approximation	743
8.6.2	The Existence and Uniqueness Theorem via Banach's Fixed Point Theorem	744
8.6.3	Hypoellipticity	745
9.	A Glance at General Scattering Theory	747
9.1	The Formal Basic Idea	749
9.2	The Rigorous Time-Dependent Approach	751
9.3	The Rigorous Time-Independent Approach	753

9.4	Applications to Quantum Mechanics	754
9.5	A Glance at Quantum Field Theory	757
9.6	Hints for Further Reading	758

Part IV. Quantum Electrodynamics (QED)

10.	Creation and Annihilation Operators	771
10.1	The Bosonic Fock Space	771
10.1.1	The Particle Number Operator	774
10.1.2	The Ground State	774
10.2	The Fermionic Fock Space and the Pauli Principle	779
10.3	General Construction	784
10.4	The Main Strategy of Quantum Electrodynamics	788
11.	The Basic Equations in Quantum Electrodynamics	793
11.1	The Classical Lagrangian	793
11.2	The Gauge Condition	796
12.	The Free Quantum Fields of Electrons, Positrons, and Photons	799
12.1	Classical Free Fields	799
12.1.1	The Lattice Strategy in Quantum Electrodynamics ..	799
12.1.2	The High-Energy Limit and the Low-Energy Limit ..	802
12.1.3	The Free Electromagnetic Field	803
12.1.4	The Free Electron Field	806
12.2	Quantization	811
12.2.1	The Free Photon Quantum Field	812
12.2.2	The Free Electron Quantum Field and Antiparticles	814
12.2.3	The Spin of Photons	819
12.3	The Ground State Energy and the Normal Product	822
12.4	The Importance of Mathematical Models	824
12.4.1	The Trouble with Virtual Photons	825
12.4.2	Indefinite Inner Product Spaces	826
12.4.3	Representation of the Creation and Annihilation Operators in QED	826
12.4.4	Gupta–Bleuler Quantization	831
13.	The Interacting Quantum Field, and the Magic Dyson Series for the S-Matrix	835
13.1	Dyson’s Key Formula	835
13.2	The Basic Strategy of Reduction Formulas	841
13.3	The Wick Theorem	846
13.4	Feynman Propagators	856

13.4.1	Discrete Feynman Propagators for Photons and Electrons	856
13.4.2	Regularized Discrete Propagators	862
13.4.3	The Continuum Limit of Feynman Propagators	864
13.4.4	Classical Wave Propagation versus Feynman Propagator	870
14.	The Beauty of Feynman Diagrams in QED	875
14.1	Compton Effect and Feynman Rules in Position Space	876
14.2	Symmetry Properties	881
14.3	Summary of the Feynman Rules in Momentum Space	882
14.4	Typical Examples	885
14.5	The Formal Language of Physicists	890
14.6	Transition Probabilities and Cross Sections of Scattering Processes	891
14.7	The Crucial Limits	894
14.8	Appendix: Table of Feynman Rules	896
15.	Applications to Physical Effects	899
15.1	Compton Effect	899
15.1.1	Duality between Light Waves and Light Particles in the History of Physics	902
15.1.2	The Trace Method for Computing Cross Sections	903
15.1.3	Relativistic Invariance	912
15.2	Asymptotically Free Electrons in an External Electromagnetic Field	914
15.2.1	The Key Formula for the Cross Section	914
15.2.2	Application to Yukawa Scattering	915
15.2.3	Application to Coulomb Scattering	915
15.2.4	Motivation of the Key Formula via <i>S</i> -Matrix	916
15.2.5	Perspectives	921
15.3	Bound Electrons in an External Electromagnetic Field	922
15.3.1	The Spontaneous Emission of Photons by the Atom	922
15.3.2	Motivation of the Key Formula	923
15.3.3	Intensity of Spectral Lines	925
15.4	Cherenkov Radiation	926

Part V. Renormalization

16.	The Continuum Limit	945
16.1	The Fundamental Limits	945
16.2	The Formal Limits Fail	946
16.3	Basic Ideas of Renormalization	947

16.3.1	The Effective Mass and the Effective Charge of the Electron	947
16.3.2	The Counterterms of the Modified Lagrangian	947
16.3.3	The Compensation Principle	948
16.3.4	Fundamental Invariance Principles	949
16.3.5	Dimensional Regularization of Discrete Algebraic Feynman Integrals	949
16.3.6	Multiplicative Renormalization	950
16.4	The Theory of Approximation Schemes in Mathematics	951
17.	Radiative Corrections of Lowest Order	953
17.1	Primitive Divergent Feynman Graphs	953
17.2	Vacuum Polarization	954
17.3	Radiative Corrections of the Propagators	955
17.3.1	The Photon Propagator	956
17.3.2	The Electron Propagator	956
17.3.3	The Vertex Correction and the Ward Identity	957
17.4	The Counterterms of the Lagrangian and the Compensation Principle	957
17.5	Application to Physical Problems	958
17.5.1	Radiative Correction of the Coulomb Potential	958
17.5.2	The Anomalous Magnetic Moment of the Electron	959
17.5.3	The Anomalous Magnetic Moment of the Muon	961
17.5.4	The Lamb Shift	962
17.5.5	Photon-Photon Scattering	964
18.	A Glance at Renormalization to all Orders of Perturbation Theory	967
18.1	One-Particle Irreducible Feynman Graphs and Divergences	970
18.2	Overlapping Divergences and Manoukian's Equivalence Principle	972
18.3	The Renormalizability of Quantum Electrodynamics	975
18.4	Automated Multi-Loop Computations in Perturbation Theory	977
19.	Perspectives	979
19.1	BPHZ Renormalization	981
19.1.1	Bogoliubov's Iterative <i>R</i> -Method	981
19.1.2	Zimmermann's Forest Formula	984
19.1.3	The Classical BPHZ Method	985
19.2	The Causal Epstein–Glaser <i>S</i> -Matrix Approach	987
19.3	Kreimer's Hopf Algebra Revolution	990
19.3.1	The History of the Hopf Algebra Approach	991

19.3.2	Renormalization and the Iterative Birkhoff Factorization for Complex Lie Groups	993
19.3.3	The Renormalization of Quantum Electrodynamics	996
19.4	The Scope of the Riemann–Hilbert Problem	997
19.4.1	The Gaussian Hypergeometric Differential Equation	998
19.4.2	The Confluent Hypergeometric Function and the Spectrum of the Hydrogen Atom	1004
19.4.3	Hilbert’s 21th Problem	1004
19.4.4	The Transport of Information in Nature	1007
19.4.5	Stable Transport of Energy and Solitons	1007
19.4.6	Ariadne’s Thread in Soliton Theory	1009
19.4.7	Resonances	1014
19.4.8	The Role of Integrable Systems in Nature	1014
19.5	The BFFO Hopf Superalgebra Approach	1016
19.6	The BRST Approach and Algebraic Renormalization	1019
19.7	Analytic Renormalization and Distribution-Valued Analytic Functions	1022
19.8	Computational Strategies	1023
19.8.1	The Renormalization Group	1023
19.8.2	Operator Product Expansions	1024
19.8.3	Binary Planar Graphs and the Renormalization of Quantum Electrodynamics	1026
19.9	The Master Ward Identity	1027
19.10	Trouble in Quantum Electrodynamics	1027
19.10.1	The Landau Inconsistency Problem in Quantum Electrodynamics	1027
19.10.2	The Lack of Asymptotic Freedom in Quantum Electrodynamics	1029
19.11	Hints for Further Reading	1029
Epilogue		1045
References		1049
List of Symbols		1061
Index		1069