
Contents

1	Trajectory Planning	1
1.1	A General Overview on Trajectory Planning	1
1.2	One-dimensional Trajectories	3
1.3	Mechanical Cams and Electronic Cams	4
1.4	Multi-dimensional Trajectories	6
1.5	Contents and Structure of this Book	8
1.6	Notation	10

Part I Basic Motion Profiles

2	Analytic Expressions of Elementary Trajectories	15
2.1	Polynomial Trajectories	15
2.1.1	Linear trajectory (constant velocity)	17
2.1.2	Parabolic trajectory (constant acceleration)	18
2.1.3	Trajectory with asymmetric constant acceleration	21
2.1.4	Cubic trajectory	23
2.1.5	Polynomial of degree five	26
2.1.6	Polynomial of degree seven	28
2.1.7	Polynomials of higher degree	30
2.2	Trigonometric Trajectories	42
2.2.1	Harmonic trajectory	42
2.2.2	Cycloidal trajectory	43
2.2.3	Elliptic trajectory	45
2.3	Exponential Trajectories	47
2.4	Trajectories Based on the Fourier Series Expansion	51
2.4.1	Gutman 1-3	53
2.4.2	Freudenstein 1-3	54
2.4.3	Freudenstein 1-3-5	55

3	Composition of Elementary Trajectories	59
3.1	Linear Trajectory with Circular Blends	59
3.2	Linear Trajectory with Parabolic Blends (Trapezoidal)	62
3.2.1	Trajectory with preassigned acceleration	65
3.2.2	Trajectory with preassigned acceleration and velocity ..	65
3.2.3	Synchronization of several trapezoidal trajectories	66
3.2.4	Trajectory through a sequence of points	67
3.2.5	Displacement time of a trapezoidal trajectory	69
3.2.6	Trajectory with assigned durations T and T_a	69
3.2.7	Trajectory with non-null initial and final velocities	70
3.3	Linear Trajectory with Polynomial Blends	76
3.4	Trajectory with Double S Velocity Profile	79
3.4.1	Computation of the trajectory for $q_1 > q_0$	88
3.4.2	Computation of the trajectory for $q_1 < q_0$	90
3.4.3	Double S with null initial and final velocities	90
3.4.4	On-line computation of the double S trajectory	93
3.4.5	Displacement time of a double S trajectory	101
3.4.6	Double S trajectory with assigned duration of the different phases	102
3.5	Fifteen Segments Trajectory	107
3.6	Piecewise Polynomial Trajectory	117
3.7	Modified Trapezoidal Trajectory	119
3.8	Modified Sinusoidal Trajectory	124
3.9	Modified Cycloidal Trajectory	127
3.10	Constant Velocity/Acceleration Trajectories with Cycloidal or Harmonic Blends	133
3.10.1	Constraints on the velocity profile	133
3.10.2	Constraints on the acceleration profile	135
3.10.3	Minimum-time trajectories	140
3.11	Trajectories with Constant Acceleration and Cycloidal/Cubic Blends	144
4	Multipoint Trajectories	151
4.1	Interpolation by Polynomial Functions	151
4.2	Orthogonal Polynomials	155
4.3	Trigonometric Polynomials	164
4.4	Cubic Splines	166
4.4.1	Computation of the coefficients for assigned initial and final velocities	169
4.4.2	Periodic cubic splines	172
4.4.3	Cubic splines with assigned initial and final velocities: computation based on the accelerations	175
4.4.4	Cubic splines with assigned initial and final velocities and accelerations	177
4.4.5	Smoothing cubic splines	180

4.4.6 Choice of the time instants and optimization of cubic splines 188

4.5 B-spline Functions for Trajectories with High Degree of Continuity 194

4.6 Nonlinear Filters for Optimal Trajectory Planning 208

4.6.1 Online trajectory planner with velocity, acceleration and jerk constraints 209

4.6.2 Online trajectory planner with velocity and acceleration constraints 216

Part II Elaboration and Analysis of Trajectories

5 Operations on Trajectories 223

5.1 Geometric Modification of a Trajectory 223

5.2 Scaling in Time 228

5.2.1 Kinematic scaling 230

5.2.2 Dynamic Scaling 236

5.3 Synchronization of Trajectories 241

6 Trajectories and Actuators 245

6.1 Trajectories and Electric Motors 245

6.1.1 Trajectories and choice of the actuator 247

6.2 Characteristics of the Motion Profiles 250

6.2.1 Comparison between trapezoidal and double S trajectories 256

7 Dynamic Analysis of Trajectories 265

7.1 Models for Analysis of Vibrations 265

7.1.1 Linear model with one degree of freedom 266

7.1.2 Linear model with n degrees of freedom 267

7.1.3 Nonlinear model with one degree of freedom 269

7.1.4 Nonlinear model with n degrees of freedom 270

7.2 Analysis of the Trajectories in the Time Domain 271

7.3 Analysis of the Trajectories in the Frequency Domain 285

7.3.1 Frequency spectrum of some elementary trajectories ... 287

7.3.2 Numerical computation of the frequency spectrum of generic trajectories 294

7.3.3 Harmonic content of periodic trajectories 299

7.3.4 Scaling and frequency properties of a trajectory 303

7.4 Frequency Modifications of Trajectories 304

7.4.1 Polydyne and splinedyne functions 305

7.4.2 Input filtering and shaping 318

7.4.3 Feedforward based on the inversion of the plant dynamics 330

Part III Trajectories in the Operational Space

8 Multidimensional Trajectories and Geometric Path Planning 341

8.1 Introduction 341

 8.1.1 Continuity of the geometric path and continuity of the trajectory 343

 8.1.2 Global and local interpolation/approximation 346

8.2 Orientation of the Tool 347

 8.2.1 Case of independent position and orientation 347

 8.2.2 Case of position and orientation coupled 353

8.3 Definition of the Geometric Path Through Motion Primitives . 356

8.4 Global Interpolation 359

 8.4.1 Definition of the set $\{\bar{u}_k\}$ 359

 8.4.2 Cubic B-spline interpolation 360

8.5 Global Approximation 364

 8.5.1 Knots choice 366

8.6 A Mixed Interpolation/Approximation Technique 368

8.7 Smoothing Cubic B-splines 371

 8.7.1 Smoothing B-splines with assigned start/end points and directions 373

8.8 B-spline Functions for Trajectories with High Degree of Continuity 376

8.9 Use of Nurbs for Trajectory Generation 391

8.10 Local Interpolation with Bézier Curves 393

 8.10.1 Computation of the tangent and curvature vectors 394

 8.10.2 Cubic Bézier curves interpolation 395

 8.10.3 Quintic Bézier curves interpolation 400

8.11 Linear Interpolation with Polynomial Blends 406

9 From Geometric Paths to Trajectories 415

9.1 Introduction 415

9.2 Constant Scaling 416

9.3 Generic Motion Law 418

9.4 Constant Feed Rate 421

9.5 Generic Feed Rate Profile 424

9.6 Integration of Geometric Path and Motion Law for Complex 3D Tasks 429

 9.6.1 Linear trajectory with polynomial blends 429

 9.6.2 B-spline trajectory 440

 9.6.3 Smoothing B-spline trajectory 445

 9.6.4 B-spline approximation of a trajectory based on motion primitives 449

Part IV Appendices

A	Numerical Issues	457
A.1	Parameters of normalized polynomials $q_N(\tau)$	457
A.2	Parameters of the Trajectory ‘4-3-4’	461
A.3	Solution of the Equation $M \mathbf{k} = \mathbf{q}$	461
A.4	Efficient Evaluation of Polynomial Functions	463
A.5	Numerical Solution of Tridiagonal Systems	464
A.5.1	Tridiagonal systems	464
A.5.2	Cyclic tridiagonal systems	465
B	B-spline, Nurbs and Bézier curves	467
B.1	B-spline Functions	467
B.1.1	B-spline basis functions	467
B.1.2	Definition and properties of B-splines	471
B.1.3	Evaluation of a B-spline curve	474
B.1.4	Derivative of a B-spline curve	475
B.1.5	Conversion from B-form to Piecewise Polynomial form (pp-form)	479
B.2	Definition and Properties of Nurbs	481
B.3	Definition and Properties of Bézier Curves	483
B.3.1	Evaluation of a Bézier curve	484
B.3.2	Derivatives of a Bézier curve	486
C	Representation of the Orientation	489
C.1	Rotation Matrices	489
C.1.1	Elementary rotation matrices	490
C.2	Angle-Axis Representation	490
C.3	Euler Angles	491
C.4	Roll-Pitch-Yaw Angles	493
D	Spectral Analysis and Fourier Transform	495
D.1	Fourier Transform of a Continuous Time Function	495
D.1.1	Main properties of the Fourier transform	496
D.2	Fourier Series of a Periodic Continuous Function	497
D.3	Fourier Transform of a Discrete Time Function	498
D.3.1	Discrete Fourier transform	499
D.4	Fourier Analysis of Signals Using DFT (and FFT)	500
	References	503
	Index	509