

Contents

1	Introduction	1
2	MISO Takagi-Sugeno Fuzzy System with Linear Membership Functions	3
2.1	Perfect Approximation of Nonlinear Functions Using the Simplest Takagi-Sugeno Model	3
2.2	Assumptions and Linguistic Interpretation of Linear Membership Functions	7
2.3	Compact Description of the MISO TS System	9
2.4	Crisp Output of the Zero-Order MISO P1-TS System	11
2.5	Completeness and Noncontradiction in Rule-Based Systems Defined by Metarules	16
2.6	Matrix Description of the MIMO Fuzzy Rule-Based System	18
2.7	Equivalence Problem in the Rule-Based Systems	20
2.8	Summary	23
3	Recursion in TS Systems with Two Fuzzy Sets for Every Input	25
3.1	Some Features of the Fundamental Matrix and Its Inverse	25
3.2	Theorem on Recursion for P1-TS Systems	27
3.2.1	Rule-Base Decomposition	28
3.2.2	Crisp Output Calculation for P1-TS System Using Recursion	29
3.3	Recursion in More General TS Systems with Two Fuzzy Sets for Every Input	31
3.4	MIMO TS Systems with Inference Concerning the Structure Parameters	38
3.5	Boundedness of P1-TS Systems	57
3.6	Summary	58

4 Fuzzy Rule-Based Systems with Polynomial Membership Functions	61
4.1 TS Systems with Two Polynomial Membership Functions for Every Input	62
4.2 The Normalized Membership Functions for P2-TS Systems	64
4.3 SISO P2-TS System	66
4.4 P2-TS System with Two and More Inputs	69
4.4.1 Rule-Base Structure for Two-Inputs-One-Output P2-TS System	71
4.4.2 Rule-Base Structure for Three-Inputs-One-Output P2-TS System	72
4.5 The Fundamental Matrix for MISO P2-TS System	73
4.6 Recursion in MISO P2-TS Systems	83
4.6.1 Rule-Base Decomposition.....	84
4.6.2 Crisp Output Calculation for P2-TS System Using Recursion	86
4.7 Recursion in More General TS Systems with Three Fuzzy Sets for Every Input	96
4.8 Summary	99
5 Comprehensive Study and Applications of P1-TS Systems	101
5.1 P1-TS Systems with Two Inputs	102
5.1.1 General Case	102
5.1.2 A Simple Controller Design for a Milk of Lime Blending Tank	103
5.1.3 P1-TS Systems with Inputs and Outputs from the Unity Interval.....	107
5.2 P1-TS Fuzzy Systems with Three Inputs	110
5.2.1 General Case	110
5.2.2 Examples of Highly Interpretable P1-TS Systems with Three Inputs	111
5.3 Examples of P1-TS Systems with Four and More Inputs	121
5.3.1 Low Order Atmospheric Circulation Model	129
5.3.2 Induction Motor Model	132
5.3.3 Acclimatization Chamber Model	137
5.4 Optimal Fuzzy Control System Design for Second Order Plant	139
5.4.1 Highly Interpretable Fuzzy Rules for PID Controller	139
5.4.2 Optimal PID Fuzzy Controller for Linear Second Order Plant	141
5.4.3 PD-Like Optimal Controller for Nonlinear Second Order Plant	143

Contents	XIX
5.5 P1-TS System as Controller with Variable Gains	148
5.6 Exact Modeling of Single-Input Dynamical Systems	151
5.7 Exact Modeling of MIMO Linear Dynamical Systems	160
5.8 Strong Triangular Fuzzy Partition	164
5.9 Linearity Condition for P1-TS Systems	174
5.10 The First-Order P1-TS Systems	175
5.11 Zero-Order TS System with Contradictory Rule-Base	177
5.12 Summary	179
6 Modeling of Multilinear Dynamical Systems from Experimental Data	183
6.1 Problem Statement	183
6.2 Problem Solution	184
6.3 Analytical Solution for Dynamical Systems with Two Variables	188
6.4 Estimation of P1-TS Model by Recursive Least Squares	195
6.5 Summary	196
7 Binary Classification Using P1-TS Rule Scheme	199
7.1 Problem Description	200
7.2 The Fuzzy Rules with Proximity Degrees	202
7.3 Binary Classifier Equation	203
7.4 P1-TS System with Similarity Degrees as Optimal Binary Classifier	210
7.5 The Regularization Algorithm and Support Vector Machines	213
7.6 Summary	215
A Kronecker Product of Matrices	217
B Generators and Fundamental Matrices for P1-TS Systems	219
B.1 Formulas for $n = 1$	219
B.1.1 Vertices of the Interval $D^1 = [-\alpha_1, \beta_1]$	219
B.1.2 Generator	219
B.1.3 Fundamental Matrix and Its Inverse	219
B.2 Formulas for $n = 2$	220
B.2.1 Vertices of the Rectangle $D^2 = [-\alpha_1, \beta_1] \times [-\alpha_2, \beta_2]$,	220
B.2.2 Generator	220
B.2.3 Fundamental Matrix and Its Inverse	220
B.3 Formulas for $n = 3$	221
B.3.1 Vertices of the Cuboid $D^3 = [-\alpha_1, \beta_1] \times [-\alpha_2, \beta_2] \times [-\alpha_3, \beta_3]$	221
B.3.2 Generator	221
B.3.3 Fundamental Matrix and Its Inverse	221

B.4	Formulas for $n = 4$	222
B.4.1	Vertices of the Hypercuboid $D^4 = [-\alpha_1, \beta_1] \times \dots \times [-\alpha_4, \beta_4]$	222
B.4.2	Generator	223
B.4.3	Fundamental Matrix and Its Inverse	223
C	Proofs of Theorems, Remarks and Algorithms	231
C.1	Proof of Remark 3.2	231
C.2	Proof of Remark 3.3	232
C.3	Proof of Corollary 5.27	233
C.4	Proof of RLS Algorithm from Section 6.4	234
References		237
Index		249