

# Contents

|  |           |
|--|-----------|
| Preface . . . . .  | vii       |
| Acknowledgments . . . . .  | viii      |
| <b>1 Introduction</b>  | <b>1</b>  |
| 1.1 The Ideal Bose Gas . . . . .   | 1         |
| 1.2 The Concept of Bose-Einstein Condensation . . . . .                        | 4         |
| 1.3 Overview and Outline . . . . .   | 6         |
| <b>2 The Dilute Bose Gas in 3D</b>   | <b>9</b>  |
| 2.1 Upper Bound . . . . .  | 14        |
| 2.2 Lower Bound . . . . .  | 17        |
| <b>3 The Dilute Bose Gas in 2D</b>   | <b>27</b> |
| <b>4 Generalized Poincaré Inequalities</b>                                     | <b>33</b> |
| <b>5 Bose-Einstein Condensation and Superfluidity for Homogeneous Gases</b>    | <b>39</b> |
| 5.1 Bose-Einstein Condensation . . . . .                                       | 39        |
| 5.2 Superfluidity . . . . .  | 43        |
| <b>6 Gross-Pitaevskii Equation for Trapped Bosons</b>                          | <b>47</b> |
| 6.1 Three Dimensions . . . . .   | 48        |
| 6.2 Two Dimensions . . . . .   | 59        |
| <b>7 Bose-Einstein Condensation and Superfluidity for Dilute Trapped Gases</b> | <b>63</b> |
| <b>8 One-Dimensional Behavior of Dilute Bose Gases in Traps</b>                | <b>71</b> |
| 8.1 Discussion of the Results . . . . .  | 75        |
| 8.2 The 1D Limit of 3D GP Theory . . . . .                                     | 78        |
| 8.3 Outline of Proof . . . . .   | 80        |

|           |   |            |
|-----------|---|------------|
| <b>9</b>  | <b>Two-Dimensional Behavior in Disc-Shaped Traps</b>                      | <b>87</b>  |
| 9.1       | The 2D Limit of 3D GP Theory . . . . .                                    | 92         |
| 9.2       | Upper Bound . . . . .   | 94         |
| 9.3       | Scattering Length . . . . .   | 101        |
| 9.4       | Lower Bound . . . . .   | 103        |
| <b>10</b> | <b>The Charged Bose Gas, the One- and Two-Component Cases</b>             | <b>109</b> |
| 10.1      | The One-Component Gas . . . . .   | 109        |
| 10.2      | The Two-Component Gas . . . . .   | 112        |
| 10.3      | The Bogoliubov Approximation . . . . .                                    | 113        |
| 10.4      | The Rigorous Lower Bounds . . . . .                                       | 116        |
| 10.5      | The Rigorous Upper Bounds . . . . .                                       | 124        |
| <b>11</b> | <b>Bose-Einstein Quantum Phase Transition in an Optical Lattice Model</b> | <b>131</b> |
| 11.1      | Introduction . . . . .  | 131        |
| 11.2      | Reflection Positivity . . . . .   | 136        |
| 11.3      | Proof of BEC for Small $\lambda$ and $T$ . . . . .                        | 137        |
| 11.4      | Absence of BEC and Mott Insulator Phase . . . . .                         | 142        |
| 11.5      | The Non-Interacting Gas . . . . .   | 147        |
| 11.6      | Conclusion . . . . .  | 147        |
| <b>A</b>  | <b>Elements of Bogoliubov Theory</b>                                      | <b>149</b> |
| <b>B</b>  | <b>An Exactly Soluble Model</b>   | <b>165</b> |
| <b>C</b>  | <b>Definition and Properties of Scattering Length</b>                     | <b>171</b> |
| <b>D</b>  | <b><math>c</math>-Number Substitutions and Gauge Symmetry Breaking</b>    | <b>177</b> |
|           | <b>Bibliography</b>   | <b>187</b> |
|           | <b>Index</b>  | <b>201</b> |