Contents

Preface **vii**

1 Introduction and Outline **1**
1.1 Description of the Contents 2
1.2 Notation and Conventions 3

2 Indefinite Inner Products **7**
2.1 Definition 7
2.2 Orthogonality and Orthogonal Bases 9
2.3 Classification of Subspaces 11
2.4 Exercises 14
2.5 Notes 18

3 Orthogonalization and Orthogonal Polynomials **19**
3.1 Regular Orthogonalizations 19
3.2 The Theorems of Szegő and Krein 27
3.3 One-Step Theorem 29
3.4 Determinants of One-Step Completions 36
3.5 Exercises 40
3.6 Notes 44

4 Classes of Linear Transformations **45**
4.1 Adjoint Matrices 45
4.2 H-Selfadjoint Matrices: Examples and Simplest Properties 48
4.3 H-Unitary Matrices: Examples and Simplest Properties 50
4.4 A Second Characterization of H-Unitary Matrices 54
4.5 Unitary Similarity 55
4.6 Contractions 57
4.7 Dissipative Matrices 59
4.8 Symplectic Matrices 62
4.9 Exercises 66
4.10 Notes 72

5 Canonical Forms **73**
5.1 Description of a Canonical Form 73
5.2 First Application of the Canonical Form 75
5.3 Proof of Theorem 5.1.1 77
5.4 Classification of Matrices by Unitary Similarity 82
5.5 Signature Matrices 85
5.6 Structure of H-Selfadjoint Matrices 89
5.7 H-Definite Matrices 91
5.8 Second Description of the Sign Characteristic 92
5.9 Stability of the Sign Characteristic 95
5.10 Canonical Forms for Pairs of Hermitian Matrices 96
5.11 Third Description of the Sign Characteristic 98
5.12 Invariant Maximal Nonnegative Subspaces 99
5.13 Inverse Problems 106
5.14 Canonical Forms for H-Unitaries: First Examples 107
5.15 Canonical Forms for H-Unitaries: General Case 110
5.16 First Applications of the Canonical Form of H-Unitaries 118
5.17 Further Deductions from the Canonical Form 119
5.18 Exercises 120
5.19 Notes 123

6 Real H-Selfadjoint Matrices **125**
6.1 Real H-Selfadjoint Matrices and Canonical Forms 125
6.2 Proof of Theorem 6.1.5 128
6.3 Comparison with Results in the Complex Case 131
6.4 Connected Components of Real Unitary Similarity Classes 133
6.5 Connected Components of Real Unitary Similarity Classes (H Fixed) 137
6.6 Exercises 140
6.7 Notes 142

7 Functions of H-Selfadjoint Matrices **143**
7.1 Preliminaries 143
7.2 Exponential and Logarithmic Functions 145
7.3 Functions of H-Selfadjoint Matrices 147
7.4 The Canonical Form and Sign Characteristic 150
7.5 Functions which are Selfadjoint in another Indefinite Inner Product 154
7.6 Exercises 156
7.7 Notes 158

8 H-Normal Matrices **159**
8.1 Decomposability: First Remarks 159
8.2 H-Normal Linear Transformations and Pairs of Commuting Matrices 163
8.3 On Unitary Similarity in an Indefinite Inner Product 165
8.4 The Case of Only One Negative Eigenvalue of H 166

8.5 Exercises 174
8.6 Notes 177

9 General Perturbations. Stability of Diagonalizable Matrices 179
9.1 General Perturbations of H-Selfadjoint Matrices 179
9.2 Stably Diagonalizable H-Selfadjoint Matrices 183
9.3 Analytic Perturbations and Eigenvalues 185
9.4 Analytic Perturbations and Eigenvectors 189
9.5 The Real Case 192
9.6 Positive Perturbations of H-Selfadjoint Matrices 193
9.7 H-Selfadjoint Stably r-Diagonalizable Matrices 195
9.8 General Perturbations and Stably Diagonalizable H-Unitary Matrices198
9.9 H-Unitarily Stably u-Diagonalizable Matrices 200
9.10 Exercises 203
9.11 Notes 205

10 Definite Invariant Subspaces 207
10.1 Semidefinite and Neutral Subspaces: A Particular H 207
10.2 Plus Matrices and Invariant Nonnegative Subspaces 212
10.3 Deductions from Theorem 10.2.4 217
10.4 Expansive, Contractive Matrices and Spectral Properties 221
10.5 The Real Case 226
10.6 Exercises 227
10.7 Notes 228

11 Differential Equations of First Order 229
11.1 Boundedness of solutions 229
11.2 Hamiltonian Systems of Positive Type with Constant Coefficients . 232
11.3 Exercises 234
11.4 Notes 236

12 Matrix Polynomials 237
12.1 Standard Pairs and Triples 238
12.2 Matrix Polynomials with Hermitian Coefficients 242
12.3 Factorization of Hermitian Matrix Polynomials 245
12.4 The Sign Characteristic of Hermitian Matrix Polynomials 249
12.5 The Sign Characteristic of Hermitian Analytic Matrix Functions . 256
12.6 Hermitian Matrix Polynomials on the Unit Circle 261
12.7 Exercises 263
12.8 Notes 266

13 Differential and Difference Equations of Higher Order 267
13.1 General Solution of a System of Differential Equations 267
13.2 Boundedness for a System of Differential Equations 268
13.3 Stable Boundedness for Differential Equations 270
13.4 The Strongly Hyperbolic Case 273
13.5 Connected Components of Differential Equations 274
13.6 A Special Case 276
13.7 Difference Equations 278
13.8 Stable Boundedness for Difference Equations 281
13.9 Connected Components of Difference Equations 284
13.10Exercises 286
13.11Notes 288

14 Algebraic Riccati Equations 289
14.1 Matrix Pairs in Systems Theory and Control 290
14.2 Origins in Systems Theory 293
14.3 Preliminaries on the Riccati Equation 295
14.4 Solutions and Invariant Subspaces 296
14.5 Symmetric Equations 297
14.6 An Existence Theorem 298
14.7 Existence when M has Real Eigenvalues 303
14.8 Description of Hermitian Solutions 307
14.9 Extremal Hermitian Solutions 309
14.10The CARE with Real Coefficients 312
14.11The Concerns of Numerical Analysis 315
14.12Exercises 317
14.13Notes 318

A Topics from Linear Algebra 319
A.1 Hermitian Matrices 319
A.2 The Jordan Form 321
A.3 Riesz Projections 332
A.4 Linear Matrix Equations 335
A.5 Perturbation Theory of Subspaces 335
A.6 Diagonal Forms for Matrix Polynomials and Matrix Functions . . . 338
A.7 Convexity of the Numerical Range 342
A.8 The Fixed Point Theorem 344
A.9 Exercises 345

Bibliography 349

Index 355