

Contents

1. Introduction	1
1.1 Essence of fuzzy set theory	1
1.2 Development of fuzzy set theory	6
2. Basic Notions of Fuzzy Set Theory	11
2.1 Fuzzy sets	11
2.2 Characteristic parameters (indices) of a fuzzy set	22
2.3 Linguistic modifiers of fuzzy sets	27
2.4 Types of membership functions of fuzzy sets	34
2.5 Type 2 fuzzy sets	52
2.6 Fuzziness and probability: two kinds of uncertainty	56
3. Arithmetic of Fuzzy Sets	59
3.1 The extension principle	60
3.2 Addition of fuzzy numbers	67
3.3 Subtraction of fuzzy numbers	75
3.4 Multiplication of fuzzy numbers	79
3.5 Division of fuzzy numbers	95
3.6 Peculiarities of fuzzy numbers	100
3.7 Differences between fuzzy numbers and linguistic values	107
4. Mathematics of Fuzzy Sets	111
4.1 Basic operations on fuzzy sets	111
4.1.1 Intersection operation (logical product) of fuzzy sets . .	112
4.1.2 Union (logical sum) of fuzzy sets	126
4.1.3 Compensatory operators	132
4.2 Fuzzy relations	136
4.3 Implication	149
5. Fuzzy Models	157
5.1 Structure, main elements and operations in fuzzy models . . .	157
5.1.1 Fuzzification	159
5.1.2 Inference	160
5.1.2.1 Premise evaluation	162

5.1.2.2	Determination of activated membership functions of conclusions in particular rules at given input values of a fuzzy model	167
5.1.2.3	Determination of the resulting membership function of the rule-base conclusion	172
5.1.3	Defuzzification of the resulting membership function of the rule-base conclusion	184
5.1.4	Example of fuzzy modeling	199
5.2	Significant features of rules, rule bases and fuzzy models	202
5.2.1	Local character of rules	202
5.2.2	Dependence of the number of rules on the number of inputs and fuzzy sets	204
5.2.3	Completeness of a fuzzy model	208
5.2.4	Consistency of the rule base	216
5.2.5	Continuity of the rule base	219
5.2.6	Redundancy of the rule base	222
5.3	Advice relating to rule base construction	224
5.4	Reduction of the rule base	229
5.5	Normalization (scaling) of the fuzzy model inputs and output	244
5.6	Extrapolation in fuzzy models	250
5.7	Types of fuzzy models	280
5.7.1	Mamdani models	281
5.7.2	Takagi-Sugeno models	301
5.7.3	Relational models	311
5.7.4	Global and local fuzzy models	316
5.7.5	Fuzzy multimodels	323
5.7.6	Neuro-fuzzy models	329
5.7.7	Alternative models	331
5.7.8	Similarity principles of the system and of the system model	338
5.7.9	Fuzzy classification	339
6.	Methods of Fuzzy Modeling	363
6.1	Fuzzy modeling based on the system expert's knowledge	366
6.2	Creation of fuzzy, self-tuning models based on input/output measurement data of the system	373
6.2.1	Application of neuro-fuzzy networks for fuzzy model parameter tuning	374
6.2.1.1	Structuring and training of neural networks	374
6.2.1.2	Transformation of a Mamdani fuzzy model into a neuro-fuzzy network	388
6.2.1.3	Transformation of a Takagi-Sugeno fuzzy model into a neuro-fuzzy network	397
6.2.2	Tuning of fuzzy model parameters with the genetic algorithm method	400

6.3 Creation of self-organizing and self-tuning fuzzy models based on input/output measurement data of the system	405
6.3.1 Determination of significant and insignificant inputs of the model	406
6.3.2 Determining of fuzzy curves	410
6.3.3 Self-organization and self-tuning tuning of fuzzy model parameters	417
6.3.3.1 Self-organization and tuning of fuzzy models with the geometric method of the maximum absolute error	420
6.3.3.2 Self-organization and self-tuning of fuzzy models with clustering methods	452
6.3.3.3 Self-organization and self-tuning of fuzzy models with the searching method	487
7. Fuzzy Control	495
7.1 Static fuzzy controllers	495
7.2 Dynamic fuzzy controllers	500
7.3 The determination of structures and parameters for fuzzy controllers (organization and tuning)	511
7.3.1 The design of fuzzy controllers on the basis of expert knowledge concerning plant under control	512
7.3.2 The design of a fuzzy controller on the basis of a model of the expert controlling the plant	516
7.3.3 The design of a fuzzy controller on the basis of the model of controlled plant	522
7.3.3.1 Remarks concerning identification of models of dynamic plants	522
7.3.3.2 Some remarks concerning the identification of inverted models of dynamical plants	525
7.3.3.3 Tuning a fuzzy controller with an a priori chosen structure	554
7.3.3.4 Fuzzy control based on the Internal Model Control Structure (IMC structure)	561
7.3.3.5 Fuzzy control structure with an inverse of a plant model (InvMC structure)	583
7.3.3.6 Adaptive fuzzy control	599
7.3.3.7 Multivariable fuzzy control (MIMO)	601
8. The Stability of Fuzzy Control Systems	609
8.1 The stability of fuzzy control systems with unknown models of plants	614
8.2 The circle stability criterion	618
8.3 The application of hyperstability theory to analysis of fuzzy system stability	625

8.3.1	The frequency domain representation of hyperstability conditions for control systems with a time invariant non-linear part	627
8.3.2	The time domain conditions for hyperstability of continuous, non-linear control systems containing a time-invariant non-linear part	655
8.3.3	The frequency domain conditions for hyperstability of discrete, non-linear control systems containing a time-invariant non-linear part	657
References	705
Index	725