

Contents

1	Introduction	1
2	Logic	5
2.1	Introduction	5
2.2	Propositional Logic	5
2.2.1	Exercises	10
2.3	Crisp Sets	12
2.3.1	Exercises	15
2.4	Fuzzy Logic	17
2.4.1	Exercises	19
3	Fuzzy Sets	21
3.1	Introduction	21
3.2	Fuzzy Sets	21
3.2.1	Exercises	29
3.3	t -norms, t -conorms	31
3.3.1	Exercises	36
3.4	Algebra of Fuzzy Sets	38
3.4.1	Exercises	40
3.5	Mixed Fuzzy Logic	42
3.5.1	Exercises	44
3.6	Alpha-Cuts	46
3.6.1	Exercises	48
3.7	Distance Between Fuzzy Sets	50
3.7.1	Exercises	52
4	Fuzzy Numbers	55
4.1	Introduction	55
4.2	Fuzzy Numbers	55
4.2.1	Exercises	60
4.3	Fuzzy Arithmetic	63
4.3.1	Extension Principle	63
	Exercises	66

4.3.2	Interval Arithmetic	68
	Exercises	69
4.3.3	Alfa-Cuts and Interval Arithmetic	71
	Exercises	74
4.3.4	Properties of Fuzzy Arithmetic	76
	Exercises	78
4.4	Fuzzy Max and Min	79
4.4.1	Exercises	82
4.5	Inequalities	84
4.5.1	Exercises	89
4.6	Defuzzification	91
4.6.1	Exercises	93
5	Fuzzy Equations	95
5.1	Introduction	95
5.2	Linear Equations	95
5.2.1	Classical Solution	96
5.2.2	Extension Principle Solution	97
5.2.3	Alfa-Cut and Interval Arithmetic Solution	99
5.2.4	Exercises	101
5.3	Other Fuzzy Equations	102
5.3.1	Exercises	106
6	Fuzzy Inequalities	109
6.1	Introduction	109
6.2	Solving $\bar{A} \cdot \bar{X} + \bar{B} \leq \bar{C}$	109
6.3	$\bar{A} \cdot \bar{X}^2 + \bar{B} \cdot \bar{X} + \bar{C} \geq \bar{D}$ (or $> \bar{D}$).	112
6.3.1	Exercises	114
7	Fuzzy Relations	115
7.1	Introduction	115
7.2	Definitions	115
7.2.1	Exercises	120
7.3	Transitive Closure	122
7.3.1	Exercises	126
7.4	Fuzzy Equivalence Relation	129
7.4.1	Exercises	132
7.5	Fuzzy Relation Equations	134
7.5.1	Exercises	138
8	Fuzzy Functions	141
8.1	Introduction	141
8.2	Extension Principle	141
8.2.1	Exercises	147
8.3	Alpha-Cuts and Interval Arithmetic	150

- 8.3.1 Exercises 153
- 8.4 Types of Fuzzy Functions 155
 - 8.4.1 Exercises 161
- 8.5 Inverse Functions 163
 - 8.5.1 Exercises 166
- 8.6 Derivatives 168
 - 8.6.1 Exercises 173
- 9 Fuzzy Plane Geometry 175**
 - 9.1 Exercises 181
- 10 Fuzzy Trigonometry 185**
 - 10.1 Introduction 185
 - 10.2 Standard Fuzzy Trigonometry 185
 - 10.2.1 Exercises 190
 - 10.3 Hyperbolic Trigonometric Functions 192
 - 10.3.1 Exercises 194
- 11 Systems of Fuzzy Linear Equations 195**
 - 11.1 Exercises 201
- 12 Possibility Theory 203**
 - 12.1 Introduction 203
 - 12.2 Discrete Possibilities 203
 - 12.2.1 Exercises 206
 - 12.3 Fuzzy Markov Chains 208
 - 12.3.1 Exercises 212
- 13 Neural Nets 215**
 - 13.1 Introduction 215
 - 13.2 Layered, Feedforward, Neural Nets 215
 - 13.2.1 Exercises 222
 - 13.3 Fuzzy Neural Nets 224
 - 13.3.1 Exercises 229
- 14 Approximate Reasoning 231**
 - 14.1 Introduction 231
 - 14.2 Approximate Reasoning 231
 - 14.2.1 Exercises 237
 - 14.3 Multiple Rules 240
 - 14.3.1 Exercises 242
 - 14.4 Discrete Case 244
 - 14.4.1 Exercises 248
 - 14.5 Other Methods 249
 - 14.5.1 Exercises 252

15 Genetic Algorithms	253
15.1 Exercises	259
16 Fuzzy Optimization	261
16.1 Introduction	261
16.2 Maximum/Minimum of Fuzzy Functions	261
16.2.1 Exercises	266
16.3 Fuzzy Problems	268
16.3.1 Exercises	274
Index	277
List of Figures	283
List of Tables	285