# Contents

**Figures, Tables, and Acronyms** **xiii**

**0 Introduction** **1**

**1 The Direct Multiple Shooting Method for Optimal Control** **13**
1.1 Problem Formulations . . . . . 13
1.2 Solution Methods for Optimal Control Problems . . . . . 18
1.3 The Direct Multiple Shooting Method for Optimal Control . . 24
1.4 Summary . . . . . 29

**2 Mixed–Integer Optimal Control** **31**
2.1 Problem Formulations . . . . . 31
2.2 Mixed–Integer Nonlinear Programming . . . . . 32
2.3 Outer Convexification and Relaxation . . . . . 41
2.4 Rounding Strategies . . . . . 48
2.5 Switch Costs . . . . . 53
2.6 Summary . . . . . 60

**3 Constrained Nonlinear Programming** **61**
3.1 Constrained Nonlinear Programming . . . . . 61
3.2 Sequential Quadratic Programming . . . . . 66
3.3 Derivative Generation . . . . . 74
3.4 Initial Value Problems and Sensitivity Generation . . . . . 80
3.5 Summary . . . . . 87

**4 Mixed–Integer Real–Time Iterations** **89**
4.1 Real–Time Optimal Control . . . . . 89
4.2 The Real–Time Iteration Scheme . . . . . 92
4.3 Contractivity of Real–Time Iterations . . . . . 99

4.4 Mixed–Integer Model Predictive Control . . . . . 103
4.5 Summary . . . . . 115

**5 Outer Convexification of Constraints 117**
5.1 Constraints Depending on Integer Controls . . . . . 117
5.2 Lack of Constraint Qualification . . . . . 120
5.3 Mathematical Programs with Vanishing Constraints . . . . . 127
5.4 An MPVC Lagrangian Framework . . . . . 131
5.5 Summary . . . . . 139

**6 A Nonconvex Parametric SQP Method 141**
6.1 SQP for Nonconvex Programs . . . . . 141
6.2 Parametric Quadratic Programs . . . . . 148
6.3 A Primal–Dual Parametric Active Set Strategy . . . . . 151
6.4 Parametric Quadratic Programming for Nonconvex Problems 163
6.5 Summary . . . . . 172

**7 Linear Algebra for Block Structured QPs 175**
7.1 Block Structure . . . . . 175
7.2 Survey of Existing Methods . . . . . 179
7.3 A Factorization for Structured KKT Systems with Many Controls . . . . . 185
7.4 Properties and Extensions . . . . . 195
7.5 Computational Complexity . . . . . 200
7.6 Summary . . . . . 203

**8 Updates for the Block Structured Factorization 205**
8.1 Matrix Updates Overview . . . . . 205
8.2 Updating the Block Local Reductions . . . . . 211
8.3 Modifying the Block Tridiagonal Factorization . . . . . 229
8.4 Summary . . . . . 235

**9 Numerical Results 237**
9.1 Mixed–Integer Optimal Control with Switch Costs . . . . . 238
9.2 Mixed–Integer NMPC Scheme Contractivity . . . . . 249
9.3 OCPs and NLPs with Vanishing Constraints . . . . . 263
9.4 Block Structured Factorization and Updates . . . . . 274
9.5 Application: A Predictive Cruise Controller . . . . . 289

**A Implementation 321**

A.1 Multiple–Shooting Real–Time Online Optimization Method . 321
A.2 Block Structured Parametric Quadratic Programming Code . 340

**Bibliography** **347**